Lec 9/9

Friday, September 9, 2016 7:55 AM

<u>Drv</u>

$*$: only for Drv

$\dagger$: for Drv and Crv

$*$ $p(x) = P(X=x)$

$*$ Conditions:

$0 \le p(x) \le 1 \quad \forall x$

$\sum_x p(x) = \sum_x P(X=x) = 1$

$\dagger$ $F(x) = P(X \le x)$ CDF - cumulative distribution function

$\dagger$ $\lim_{x \to -\infty} F(x) = 0$

$\dagger$ $\lim_{x \to \infty} F(x) = 1$

$\dagger$ F nondecreasing

$*$ F right-continuous step function

§3.3 and 3.4 CRVs

measuring something.

In practice, we tend to round these numbers.

↳ this makes distributions discrete

$[0, 10] \to \{0, 1, 2, \ldots, 10\}$

$\{0.0, 0.1, \ldots, 9.9, 10\}$

$\{0.00, 0.01, \ldots, 9.99, 10\}$

lim as # decimal places goes to infinity

for a DRV we have a pmf: probability <u>mass</u> function $p(x) = P(X=x)$

" CRV " pdf " <u>density</u> " so $\int_a^b f(x)\,dx = P(a \le X \le b)$

Conditions:

1) $f(x) \ge 0 \quad \forall x \in \mathbb{R}$

2) $\int^{\infty} f(x)\,dx = 1$

2) $\int_{-\infty} f(x)\,dx = 1$

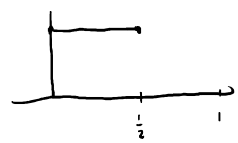
$f(x)$ can be >1 for some x so $f(x) \neq P(X = x)$

For CRV X w/ pdf $f(x)$, $P(a \le X \le b) = \int_a^b f(x)\,dx$ $\Rightarrow$ Prob is area

$P(X=c)$ $\int_c^c f(x)\,dx = 0$ $\forall c \in \mathbb{R}$

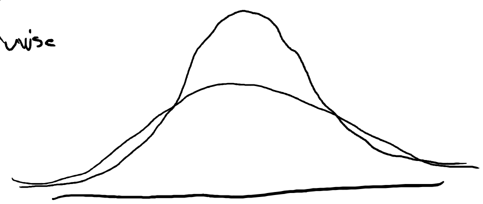
$\Rightarrow$ $P(X \in [a,b]) = P(X \in (a,b)) = \dots$

Let $f(x) = \begin{cases} 2 & 0 \le x \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$



Let $f(x) = \begin{cases} 2e^{-x/2} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$

normal distribution



$\underline{Ex}$: Let $f(x) = \begin{cases} kx^2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

find k so $f(x)$ is a valid pdf.

So $k \geq 0$ by 1).

now $\int_0^2 kx^2\,dx = \frac{k}{3} x^3 \Big|_0^2$

$= k\frac{8}{3}$

$= 1$

so $k = \frac{3}{8}$

With DRV, CDF $F(x) = P(X \le x) = \sum_{i \le x} p(i)$

CRV, CDF $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$

$\lim_{x \to -\infty} F(x) = 0$

$\lim_{x \to \infty} F(x) = 1$

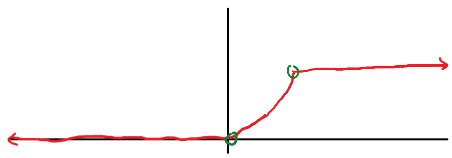
F nondecreasing & continuous

Ex: find $F(x)$ for f in the previous example.

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x > 2 \\ \frac{x^3}{8} & \text{if } x \in [0,2] \end{cases}$$

Let $x \in [0,2]$

then $F(x) = \frac{3}{8}\int_0^x t^2\,dt = \left.\frac{t^3}{8}\right|_0^x = \frac{x^3}{8}$



non-diffable

but that's ok because

$$\frac{d}{dx}F(x) = \frac{3}{8}x^2 = f(x) \quad \text{if } 0 < x < 2$$

$$= 0 = f(x) \quad \text{otherwise}$$

☆ $\frac{d}{dx}F(x) = f(x)$ where this derivative exists

and where its non-diffable simply let it $= 0$.

$$P(a \le X \le b) = F(b) - F(a) = P(X \le b) - P(X \le a)$$