Lec 9/7

Wednesday, September 7, 2016 8:01 AM

Random Vars: (capitals)

$X: S \to \mathbb{R}$

$P(X = x)$

RV — Real Num

two types:

Discrete - listable

Continuous - unlistable (measures)

X = gender of randomly selected student

$X = 0$ if male, $X = 1$ if female

$P(X = 1) = P(\text{a female is chosen})$

Ex: discrete or cts?

1) X = # of free throws made in n shots

Discrete. $\text{im}X = \{0, 1, \ldots, n\}$ (Binomial)

2) W = wait time in whole minutes of a call (Poisson) → "fish"

Discrete. $\text{im}W = \{0, 1, \ldots\}$

3) T = wait time at a bus stop (cabs)

Continuous. $\text{im}T = (0, 8)$ or $[0, 8]$ (exponential) ← if it was $[0, \infty)$

4) Y = duration of a movie trailer

Continuous $\text{im}Y = (0, \infty)$ or $(0, 120)$

Goal: determine probability distribution of an RV (CRV, DRV)

DRV first

Probability dist for a DRV lists all possible values and the probability that the DRV takes on those values.

Prob Dist . table

. formula

$p(x) := P(X = x)$ Probability mass function. (pmf)

Properties of pmf:

- $0 \leq p(x) \leq 1 \quad \forall x \in \text{im}X$
- $\sum_{x \in \text{im}X} p(x) = 1$

Ex: Sum of two fair dice

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Ex: X = sum of 2 fair dice

mutually exclusive

$$P(\text{roll a } 2, 3, \text{ or } 12) = P(X=2) + P(X=3) + P(X=12)$$
$$= \frac{1}{36} + \frac{2}{36} + \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$$

CDF: cumulative distribution function

$$F(x) := P(X \leq x) = \sum_{\substack{t \in \text{im}X \\ t \leq x}} P(X=t) \qquad \forall x \in \mathbb{R}$$

1) nondecreasing function
2) $\lim_{x \to -\infty} F(x) = 0$
3) $\lim_{x \to \infty} F(x) = 1$
4) $F(x)$ is right continuous

from the dice example:

$F(x) = 0 \quad \forall x < 2$

$F(x) = 0 \quad \forall x < 2$

$F(2) = \frac{1}{36} = P(X < 2) + P(2)$

$F(x) = \frac{1}{36} \quad \forall x \text{ s.t. } 2 \leq x < 3$