Lec 9/23

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X an RV, g a function

$$E(g(X)) = \sum_x g(x) P(X=x)$$

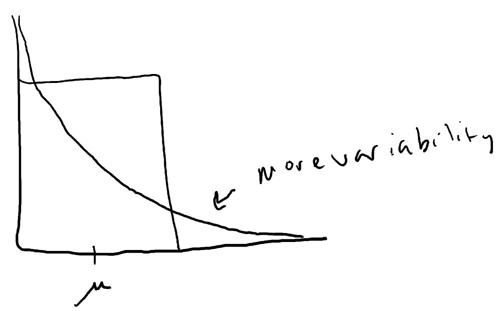
$$\text{or} = \int_{\mathbb{R}} g(x) f(x)\, dx$$

rth moment of X is $\mu'_r = E(X^r)$

rth moment about the mean/central moment is $\mu_r = E((X-\mu)^r)$

$r \in \mathbb{N}$

$$\mu_2 = E((X-\mu)^2) = \sigma^2 = Var(X) = V(X) = \sigma_X^2$$



Central moments give info about the shape of the distribution.

$\sqrt{\sigma^2} = \sigma$, the standard deviation $\sigma_X = SD(X) = \sigma$

$$Var(X) = \sum_x (x-\mu)^2 P(X=x)$$

$$\text{or} = \int_{\mathbb{R}} (x-\mu)^2 f(x)\, dx$$

$$\Downarrow$$

$$Var(X) = E(X^2) - \mu^2 = E(X^2) - [E(X)]^2$$

W: winnings in roulette on black

-1	1
$\frac{20}{38}$	$\frac{18}{38}$

$E(W) = -0.052$

$E(W^2) = 1$

$Var(W) = 1 - (-0.052)^2 = 0.997$ dollars squared

$SD(W) = \sqrt{Var(W)} = \$0.999$

W': winnings on \$1 bet on a single #

-1	35
$\frac{37}{38}$	$\frac{1}{38}$

$E(W') = -0.052$

$E(W'^2) = \frac{37}{38} + \frac{35^2}{38} =$

$$Var(W') - Var(W) = E(W'^2) - E(W^2)$$

Ex: $f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{o.w.} \end{cases}$

$E(X) = 3/4 \quad E(X^2) = \frac{3}{5} \qquad Var(X) = \frac{3}{5} - \frac{9}{16} = \frac{3}{80}$

$$E(aX + b) = aE(X) + b$$

if a, b consts, X an RV then

$$Var(aX+b) = a^2 Var(X)$$

Let $T = W_1 + W_2 + \dots + W_{10}$

$$E(10W) = E(T)$$

but

$$Var(10W) = 10\, Var(T)$$

4.5 moment generating functions

$\mu_1' = \mu = E(X)$

$\mu_2' = E(X^2)$

Can be obtained by definition (integration or summing)

but need $f(x)$ or $p(x)$

<u>Def:</u> mgf of X is given by

$$M_X(t) = E(e^{tx}) = \sum_x e^{tx} p(x)$$

$$\text{or} = \int_{\mathbb{R}} e^{tx} f(x)\, \partial x$$

if it exists.

(t in some neighborhood about 0)

$(t \in (-\varepsilon, \varepsilon))$