Lec 9/21

Wednesday, September 21, 2016 8:06 AM

$$E(g(x)) = \sum g(x)\, p(x) \quad \text{or} \quad \int g(x) f(x)$$

<u>Ex</u> $f(x) = \begin{cases} 3x^2 & 0 < x < 1 \\ 0 & \text{o.w} \end{cases}$

$$E(X) = \frac{3}{4}$$

$$E(X^2) = \frac{3}{5}$$

$a, b \in \mathbb{R}$

$$E(aX + b) = aE(X) + b \qquad \text{(proof on board)}$$

Note: $E(aX) = aE(X)$

$E(X+Y) = E(X) + E(Y)$

$E(b) = b$

but $E(X^2) \neq (E(X))^2$

in general, $E(g(X)) \neq g(E(X))$

let X be a RVs, $g_i(X)$ e functions of X s.

then $E\left(\sum_i g_i(X)\right) = \sum_i E(g_i(X))$

Proof:

$$E\left(\sum_i g_i(X)\right) = \sum_x \left[\sum_i g_i(x)\right] P(X = x)$$

$$= \sum_x \left[\sum_i g_i(x) P(X=x)\right]$$

$$= \sum_i \left[\sum_x g_i(x) P(X=x)\right]$$

$$= \sum_i E(g_i(X)) \qquad \blacksquare$$

Let X be RV, g_i functions, c_i constants

then $\underbrace{\sum_i c_i g_i(X)}_{\text{a RV}}$ is a linear combination of $g_i(X)$ s.

$$E\left(\sum_i c_i g_i(X)\right) = \sum_i c_i E(g_i(X))$$

If $X_1, \ldots, X_n$ are RVs w. joint pdf/pmf $f/p\,(x_1, \ldots, x_n)$

and g is a func of $X_1, \ldots, X_n$

$$E(g(X_1, \ldots, X_n)) = \sum_{x_1} \cdots \sum_{x_n} g(x_1, \ldots, x_n)\, p(x_1, \ldots, x_n)$$

$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} g(x_1, \ldots, x_n) f(x_1, \ldots, x_n)\, dx_n \cdots dx_1$$

Ex

$Y \backslash X$	0	1
0	0.38	0.17
1	0.14	0.07
2	[illegible]	0.05

$$E(X+Y) = \sum_{x=0}^{1} \sum_{y=0}^{2} (x+y)\, p(x,y)$$

$$= 0(0.38) + 1(0.14) + 2(0.24)$$

$$+ 1(0.17) + 2(0.07) + 3(0.05)$$

Y		
1	0.14	0.02
2	.24	0.05

$$= 0(0.38) + 1(0.14) + 2(0.24) + 1(0.17) + 2(0.02) + 3(0.05)$$

$$= 0.14 + 0.48 + 0.17 + 0.04 + 0.15$$

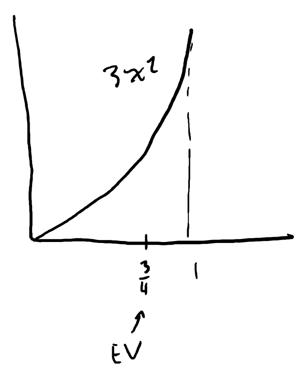
$$= 0.98$$

$\underline{Ex}$: $f(x,y) = \begin{cases} 3x & \text{if } 0 < y < x < 1 \\ 0 & \text{o.w} \end{cases}$

$$E(XY) = \iint_{\mathbb{R}^2} xy\, f(x,y)\, dy\, dx = \int_0^1 \int_0^x (xy)\, 3x \, dy\, dx = \frac{3}{2}\int_0^1 x^4\, dx = \frac{3}{10}$$

If $X_1, \dots, X_n$ are RVs, g_i funcs, c_i consts.

then $$E\left(\sum_i c_i g_i(X_1, \dots, X_n)\right) = \sum_i c_i\, E g_i(X_1, \dots, X_n)$$

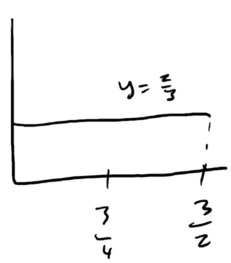


-1	1
$\frac{20}{38}$	$\frac{18}{38}$

-1	35
$\frac{37}{38}$	$\frac{1}{38}$

$P(\text{lose 20 in a row})$
$$= \left(\frac{20}{38}\right)^{20} = 0.000002$$

$P(\text{lose 20 in a row})$
$$= \left(\frac{37}{38}\right)^{20} = 0.5$$



EV but more variable.

4.3 Moments

Let X be a RV w/ pmf/pdf p/f

then the r^{th} moment of the RV X

is denoted by μ_r' and is given by

$$E(X^r) = \sum_x x^r p(x)$$

Note: $r = 0, 1, \dots$

$$\text{or} \quad = \int_{-\infty}^{\infty} x^r f(x)$$

if $r = 0$ then $\mu_0' = E(X^0) = 1$

$1 \quad \mu_1' = E(X) = \mu$

the r^{th} central moment (aka r^{th} moment about the mean)

is $\mu_r = E((X-\mu)^r) = E((X-E(X))^r)$

If $r=2$, $E((X-\mu)^2) =$ variance of X