Lec 9/2

Friday, September 2, 2016 8:01 AM

Recall: if $P(B) > 0$

Then $P(A|B) = \frac{P(A \cap B)}{P(B)} \Leftrightarrow P(B)P(A|B) = P(A \cap B)$

A, B ind $\Leftrightarrow P(A \cap B) = P(A)P(B) \Leftrightarrow P(A) = P(A|B) \Leftrightarrow P(B) = P(B|A)$

A, B ind $\Leftrightarrow A', B'$ ind (Proof on pg 3)

Ex

A: "draw a red card"

B: "draw a face card"

C: "draw a 10 or higher"

A, B ind:

$P(A \cap B) = \frac{6}{52}$ $P(A)$ $\frac{26}{52}$ $P(B) = \frac{12}{52}$

$P(A)P(B) = P(A \cap B)$ ✓

B, C not ind:

$P(C|B) = 1$ $P(C) \neq 1$

$P(B|C) = \frac{12}{20} = 0.6$ $P(B) = \frac{12}{52} \neq 0.6$ ✓ B, C dependent

Independence for >2 events:

Let $A_1, A_2, \ldots A_k \subseteq S$

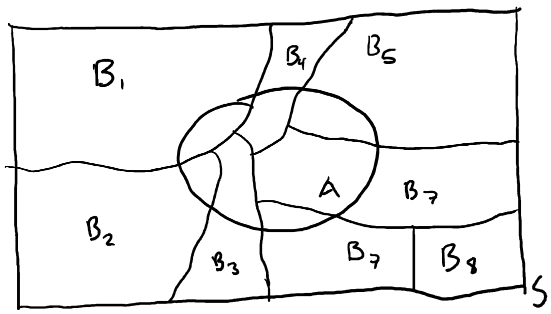
They're said to be mutually independent iff any subset of $\{A_1, \ldots, A_k\}$ is a set of independent events.

So they must be pairwise independent

Baye's Theorem

Cond. probability you have is not the one you want.

$P(+|D)$ $\quad P(D|+) < 1$



$A \subseteq S$,

$B_1, \ldots, B_K$ a partition of S

so $\forall i,j \ (i \neq j \Rightarrow B_i \cap B_j = \emptyset)$ and $\bigcup_{i=1}^{K} B_i = S$

$$P(A) = \sum_{i=1}^{K} P(A \cap B_i)$$

$$= \sum_{i=1}^{K} P(B_i) P(A \mid B_i)$$

Ex: Someone has a home phone, cell phone, & office phone

they get calls with probability 0.15, 0.45, and 0.4 respectively

% calls from telemarketers are 0.5, 0.3, and 0.1 respectively

Probability that a randomly selected call from a telemarketer

T = call is from tmarketer

O = call is to office

C = cell

H = home

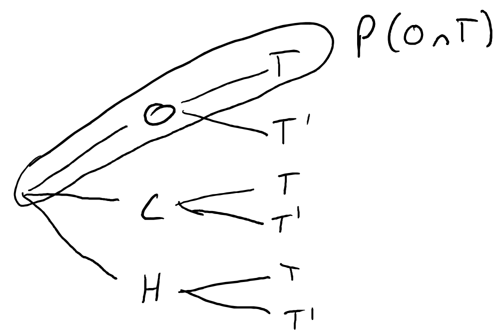
$\{O, C, H\}$ is a partition of all phone calls

$$P(T) = P(O)P(T|O) + P(C)P(T|C) + P(H)P(T|H)$$

$$= 0.4 \cdot 0.1 + 0.45 \cdot 0.3 + 0.15 \cdot 0.5$$

$$= 0.04 + 0.135 + 0.075$$

$$= 0.25$$



Baye's Theorem:

Let $B_1, \dots, B_k$ be a partition of S s.t. $P(B_i) \neq 0 \ \forall i$

Then $\forall A \subseteq S$

$$P(B_i \mid A) = \frac{P(A \cap B_i)}{P(A)} = \frac{P(B_i)\,P(A \mid B_i)}{\sum_{j=1}^{k} P(B_j)\,P(A \mid B_j)}$$

$$\underbrace{P(D \mid +)}_{D \to B_1,\ + \to A} = \frac{P(D)\,P(+ \mid D)}{\underbrace{P(D)}_{\text{how prevalent is D}}\,\underbrace{P(+ \mid D)}_{\text{true positive proportion}} + P(D')\,\underbrace{P(+ \mid D')}_{\text{false positive proportion}}}$$

$P(B_i)$ is called the "prior" probability

$P(B_i \mid A)$ " " "posterior" "

<u>Ex</u> Is surveillance justified?

Suppose a system identifies a future terrorist correctly with probability 0.99

" " non terrorist correctly " 0.999

Suppose there are 2,000 future terrorists in the US (pop 320,000,000)

1 person is selected at random and classified as a future terrorist by this system

what is the probability that the system was correct.

$$P(T \mid C) = \frac{P(T)\,P(C \mid T)}{P(T)\,P(C \mid T) + P(T')\,P(C \mid T')}$$

$$= \frac{\frac{2000}{320\,000\,000}(0.99)}{\frac{2000}{320000000}(0.99) + \left(1 - \frac{2000}{320000000}\right)(0.001)} = 0.00615$$