Lec 9/19

Monday, September 19, 2016 7:59 AM

if $X_1, \dots, X_n$ RVs. pdf: $f(x_1, \dots, x_n)$

They are mult. ind. if $\exists\, \omega_1, \dots \omega_n$

so that $f(x_1, \dots, x_n) = \omega_1(x_1) \cdot \dots \cdot \omega_n(x_n)$

$$f(x) = \begin{cases} \dots & \text{if } 0 \le x \le y \le 1 \\ \dots & \end{cases}$$

impossible to factor stuff like this.

Ex:

if x_1, x_2, x_3 are ind w/

$$f(x_1) = \begin{cases} 1 & 0 \le x_1 \le 1 \\ 0 & \text{o.w.} \end{cases}$$

$$f(x_2) = \begin{cases} e^{-x_2} & x_2 > 0 \\ 0 & \text{o.w.} \end{cases}$$

$$f(x_3) = \begin{cases} \frac{3}{8} x_3^2 & 0 \le x_3 \le 2 \\ 0 & \text{o.w} \end{cases}$$

find $f(x_1, x_2, x_3)$ and $P(X_1 + X_3 \le 1, X_2 > k)$

for $k > 0$

Soln:

$f(x_1, x_2, x_3) = \Big\{ \frac{3}{8} e^{-x_2} x^2 \quad 0 \le x_1 \le 1 \quad x_2 \geq 0, \quad 0 \le x_3 \le 2$

$$f(x_1, x_2, x_3) = \begin{cases} \frac{3}{8} e^{-x_2} x_3^2 & 0 \le x_1 \le 1, \quad x_2 \geq 0, \quad 0 \le x_3 \le 2 \\ 0 & \text{o.w.} \end{cases}$$

$$P(X_1 + X_3 \le 1,\ X_2 > k) = P(X_1 \le 1 - X_3,\ X_2 > k)$$

$$= \int_0^k \int_0^1 \int_0^{1-x_3} \frac{3}{8} e^{-x_2} x_3^2 \, dx_1 \, dx_3 \, dx_2$$

$$= \int_k^\infty \frac{3}{8} e^{-x_2} dx_2 \int_0^1 (x_3^2 - x_3^3) \, dx_3$$

$$= e^{-k}/32 \qquad \text{for } k > 0$$

Ch 4 Mathematical Expectation §4.2 EV for RV

for a drv, the expected value is a weighted avg of the possible values w/ the probabilities of those values as the weights.

x	0	2
$P(X=x)$	$\frac{1}{2}$	$\frac{1}{2}$

$$EV = \frac{\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 2}{1} = 1$$

"expected value"

If X is a DRV then $E(X) = \mu_X = \sum_x \overbrace{x P(X=x)}^{p(x)}$

If X is a CRV then $E(x) = \mu_X = \int_{-\infty}^{\infty} x f(x)\, dx$

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 1 \\ 0 & \text{o.w.} \end{cases} \qquad E(x) = \int_{-\infty}^{\infty} x f(x)\, dx = \int_0^1 x\, dx = \frac{1}{2}$$

Ex: $\underbrace{0, 00}_{G}, \underbrace{1, 2, \dots, 36}_{\frac{1}{2} B \ \frac{1}{2} R}$ (roulette)

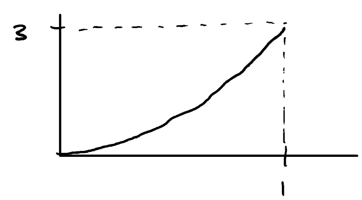
A \$1 wager on Black pays 1 to 1

W = winnings on a \$1 wager on black

w	-1	1
$P(W=w)$	$\frac{20}{38}$	$\frac{18}{38}$

$$\Rightarrow E = \frac{-2}{38} = \frac{-1}{19} = -\$0.052$$

Ex $g(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{o.w.} \end{cases}$



$\underline{Ex}$ $g(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{o.w.} \end{cases}$

$$E(X) = \int_0^1 x(3x^2)\,dx = \frac{3}{4}$$

EV of a func of a RV. e.g. $E(Y)$ where $Y = g(x)$

$$E[g(X)] = \sum_x g(x)\, P(X=x) \qquad \text{if DRV}$$

$$= \int_{-\infty}^{\infty} g(x) f(x)\,dx \qquad \text{if CRV}$$

$$E(W^2) = 1$$

$$E(X^2) = \int_0^1 x^2 (3x^2)\,dx = \frac{3}{5}$$