Lec 9/16

Friday, September 16, 2016 8:00 AM

Marginal dist: pmf of X given pmf of X, Y (or more vars)

Conditional: $\dfrac{f(x,y)}{f(x)} = f(y|x)$

$$f(x,y) = \begin{cases} 3x & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{o.w} \end{cases}$$

$$f(x|y) = \frac{2x}{1-y^2}$$

Ex $P(X > \frac{1}{2} \mid Y = \frac{1}{3}) = \int_{\frac{1}{2}}^{1} \frac{9x}{4}\,dx = \frac{9}{8} - \frac{9}{32} = \frac{25}{32}$

if $y = \frac{1}{3}$, $f(x \mid Y = \frac{1}{3}) = \frac{9x}{4}$ for $\frac{1}{3} \le x \le 1$

$$P(X \ge \tfrac{1}{4} \mid Y = \tfrac{1}{3}) = \int_{\frac{1}{4}}^{1} f(x \mid Y = \tfrac{1}{3})\,dx = \int_{\frac{1}{4}}^{\frac{1}{3}} 0\,dx + \int_{\frac{1}{3}}^{1} \frac{9x}{4}\,dx$$

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if $X_1, \dots, X_n$ are RVs, w/ joint pdf/pmf

$p(x_1, \dots, x_n)$ / $f(x_1, \dots, x_n)$

you can find any marginal or conditional:

$$f(x_1, x_2 \mid x_3, \dots, x_n) = \frac{f(x_1, \dots, x_n)}{f(x_3, \dots, x_n)}$$

$$p(x_3, \dots, x_n \mid x_1, x_2) = \frac{p(x_1, \dots, x_n)}{p(x_1, x_2)}$$

$$f(x_1, x_2 \mid x_3) = \frac{f(x_1, x_2, x_3)}{f(x_3)}$$

$$f(x_1, x_2, x_3) = \int^{\infty} \cdots \int^{\infty} f(x_1, \dots, x_n)\, dx_4 \cdots dx_n$$

$$f(x_1, x_2, x_3) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1, \dots, x_n)\, dx_4 \cdots dx_n$$

Independence

the conditional distribution of X given Y may not actually depend on y

note: $\begin{cases} \text{this depends on } y: & f(x) = 3x^2 \text{ if } 0 < y < x < 1 \\ \text{this does not}: & f(x) = 3x^2 \text{ if } 0 < x < 1 \end{cases}$

In this case, $f(x|y) = f(x) \Rightarrow X, Y$ ind

- If $X_1, \dots X_n$ are RVs w/ joint pmf/pdf $p(x_1, \dots, x_n) / f(x_1, \dots, x_n)$ and marginal of X_i is $p(x_i)$ or $f(x_i)$, then X_i's are mutually independent iff $p(x_1, \dots, x_n) = p(x_1) \cdot \dots \cdot p(x_n)$

  or $f(x_1, \dots, x_n) = f(x_1) \cdot \dots \cdot f(x_n)$

- If $X_1, \dots X_n$ are RVs w/ joint pmf/pdf $p(x_1, \dots, x_n) / f(x_1, \dots, x_n)$ Then X_is are mutually independent iff

  $$p(x_1, \dots, x_n) = \omega_1(x_1) \cdot \dots \cdot \omega_i(x_i) \cdot \dots \cdot \omega_n(x_n)$$

  or $$f(x_1, \dots, x_n) = w_1(x_1) \cdot \dots \cdot w_n(x_n)$$

  Where $\omega_i(x_i)$ only depends on x_i and $w_i(x_i) \geq 0 \ \forall x_i \in \mathbb{R}$

  (removing the restriction of pmfs/pdfs)

  let $\sum_{x_i} w_i(x_i) = c$ now $p_i(x_i) = \frac{\omega_i(x_i)}{c}$ to get a valid pmf

Ex: $f(x, y) = \begin{cases} 2x & \text{if } 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{o.w} \end{cases}$

x and y are independent

$$f(x,y) = \omega_1(x)\,\omega_2(y) = (2x)(1)$$

$$\omega_1(x) = \begin{cases} 2x & 0<x<1 \\ 0 & \text{o.w} \end{cases}$$

$$\omega_2(y) = \begin{cases} 1 & 0<y<1 \\ 0 & \text{o.w} \end{cases}$$

<u>Ex:</u> $$f(x,y) = \begin{cases} 3x & 0<y<x<1 \\ 0 & \text{o.w} \end{cases}$$

X, Y not ind.

$$f(x,y) = (3x)(1) = \omega_1(x)\,\omega_2(y)$$

but $$\omega_1 = \begin{cases} 3x & y<x<1 \\ 0 & \text{o.w.} \end{cases}$$

$$\omega_2 = \begin{cases} 1 & 0<y<x \\ 0 & \text{o.w} \end{cases}$$

not purely reliant on only one var.
not independent.

note: indicator functions:

$I(\text{cond}) = 1$ if cond is true, 0 else.

so $f(x,y) = 3x\, I(0<y<x<1)$

and in the previous example:

$$f(x,y) = 2x\, I(0<x<1)\, I(0<y<1)$$

so $\omega_1(x) = 2x\, I(0<x<1)$

Note: if you're not on a product space, the RVs are not independent.

Ex: 1f