Lec 9/14

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X, Y DRV

$p(x,y) = P(X=x, Y=y)$

is the joint pmf

$F(x,y) = P(X \le x, Y \le y)$

$= \sum_{i \le x} \sum_{j \le y} p(i,j)$

is the joint cdf

X, Y CRV

$f(x,y)$ is the joint pdf

$P((X,Y) \in A) = \iint_A f(x,y)\,dy\,dx$

$F(x,y) = P(X \le x, Y \le y)$

$= \int_{-\infty}^{x} \int_{-\infty}^{y} f(i,j)\,dj\,di$

is the joint cdf

These ideas extend to n RVs (D or C)

Marginal Distributions §3.6

$X = \begin{cases} 0 & \text{if child survived} \\ 1 & \text{if not} \end{cases}$

$Y = \begin{cases} 0 & \text{if no seat belt} \\ 1 & \text{if adult seat belt} \\ 2 & \text{if car seat belt} \end{cases}$

Y \ X	0	1	total
0	.38	.17	0.55
1	.14	.02	0.16
2	.24	.05	0.29
total	0.76	0.24	

pmf of X

marginal distribution of X

pmf of Y

marginal distribution of Y

Get prob. dist. for Y:

If X, Y are DRVs w/ joint pmf $p(x,y)$, then the marginal dists of Y and X are as follows:

$p_Y(y) = \sum_x p(x,y)$ $\qquad p_X(x) = \sum_y p(x,y)$

If X, Y are CRVs w/ joint pdf $f(x,y)$ then the marginal dists of X and Y are as follows:

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

Ex: $f(x,y) = \begin{cases} 3x & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{o.w.} \end{cases}$

find marginal distributions of X and Y

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^x 3x\,dy = 3xy\Big|_0^x = 3x^2 \quad \text{so} \quad f_X(x) = \begin{cases} 3x^2 & \text{for } 0 \le x \le 1 \\ 0 & \text{o.w.} \end{cases}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_y^1 3x\,dx = \frac{3}{2}x^2\Big|_y^1 = \frac{3}{2} - \frac{3}{2}y^2 \quad \text{so} \quad f_Y(y) = \begin{cases} \frac{3}{2} - \frac{3}{2}y^2 & \text{for } 0 \le y \le 1 \\ 0 & \text{o.w.} \end{cases}$$

Start w/ joint pmf or pdf for n RVs. you can find the marginal distribution of any subset of n RVs by summing/integrating out the rest of the n RVs.

$f(x_1, x_2, \dots, x_n)$

$$f(x_1, x_3, x_7) = \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}\, f(x_1, \dots, x_n)\, \underbrace{dx_2 \dots dx_n}_{\text{not including } x_1,\, x_3,\, \text{or } x_7}$$

$p(x_1, x_2, \dots, x_n)$

$$p(x_1, x_2) = \sum_{x_3} \cdots \sum_{x_n} p(x_1, \dots, x_n)$$

Conditional Distributions §3.7

Recall: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Now, look at DRVs X and Y. let $A := (X = x)$ and $B := (Y = y)$

$$P(X = x \mid Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{p(x,y)}{p_Y(y)}$$

$$P(X=x \mid Y=y) = \frac{P(X=x,\ Y=y)}{P(Y=y)} = \frac{p(x,y)}{p_Y(y)}$$

$\parallel$

$\underbrace{p(x \mid y)}_{\text{bad notation}}$

Cond dist: $\dfrac{\text{joint dist}}{\text{marginal dist}}$

Formally, if X, Y are DRVs with joint pmf $p(x,y)$ and marginal pmfs $p_X(x)$ and $p_Y(y)$ then cond. dists. are given by:

$$p(x \mid y) = \frac{p(x,y)}{p_Y(y)} \qquad p(y \mid x) = \frac{p(x,y)}{p_X(x)}$$

valid as long as you're not dividing by 0.

Ex: accident/child/seatbelt example

Cond. dist of Y given $X=0$

$$p(0 \mid 0) = P(Y=0 \mid X=0) = \frac{p(0,0)}{p_X(0)} = \frac{0.38}{0.76} = 0.5$$

$$p(1 \mid 0) = P(Y=1 \mid X=0) = \frac{p(0,1)}{p_X(0)} = \frac{0.14}{0.76} = 0.1842$$

$$p(2 \mid 0) = P(Y=2 \mid X=0) = \frac{p(0,2)}{p_X(0)} = \frac{0.24}{0.76} = 0.3158$$

If X, Y CRVs w/ joint pdf $f(x,y)$ and marginal distributions $f_X(x)$, $f_Y(y)$, the cond dists are given by:

$$f(x \mid y) = \frac{f(x,y)}{f_Y(y)} \qquad f(y \mid x) = \frac{f(x,y)}{f_X(x)}$$

Ex: $f(x,y) = \begin{cases} 3x & \text{if } 0 \le y \le x \le 1 \\ 0 & \text{o.w} \end{cases}$

$f_X(x) = \begin{cases} 3x^2 & \text{if } 0 \le x \le 1 \\ 0 & \text{o.w} \end{cases}$

$f_Y(y) = \begin{cases} \frac{3}{2} - \frac{3}{2}y^2 & \text{if } 0 \le y \le 1 \\ 0 & \text{o.w} \end{cases}$

Combine restrictions of two functions you're using

$$- \quad \frac{3x}{} \quad - \quad \frac{2x}{} \quad \text{if } 0 \le y < x \le 1$$

of two functions you're using

$$f(x|y) = \frac{3x}{\frac{3}{2}(1-y^2)} = \frac{2x}{1-y^2} \quad \text{if} \quad 0 \le y < x \le 1$$

$$f(y|x) = \frac{3x}{3x^2} = \frac{1}{x} \quad \text{if} \quad 0 < y \le x \le 1$$