# Lec 9/12

Monday, September 12, 2016 7:59 AM

$f(x)$ a pdf

$$P(a \leq X \leq b) = \int_a^b f(x)\,dx$$

$F(x)$ a cdf

$$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\,dx$$

$$\frac{d}{df} F(x) = f(x)$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Ex: let $Y$ be a CRV w/

pdf $f(y) = \begin{cases} e^{-y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$

1) show $f(y)$ is a valid pdf

2) find $F(y)$

1) a) $e^{-y} > 0 \quad \forall y \in \mathbb{R}, \quad 0 \geq 0,$

so $f$ is a valid pdf

B) $\int_{-\infty}^{\infty} f(y)\,dy = \int_0^{\infty} e^{-y}\,dy = -e^{-y}\Big|_0^{\infty} \quad 0 - (-1) = 1$

2) $F(y) = \int_{-\infty}^{y} f(t)\,dt = \int_0^y e^{-t}\,dt = -e^{-t}\Big|_0^y = -e^{-y} - (-1) = 1 - e^{-y}$ for $y > 0$

$$F(y) = \begin{cases} 1 - e^{-y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

F satisfies conditions of a CDF.

## Multivariate Distributions 3.6

Two or more RVs varying together

↓ bivariate ↓ multivariate

$X_1 :=$ Smoker? (Y/N)
$X_2 :=$ BMI cts
$X_3 :=$ SBP cts

} events dependent on each other.

Look at the joint distribution:

Ex: Consider joint prob dist associated w/ fatal auto accidents in which a child under age 5 was in the car and what type of seatbelt is used.

$$X = \begin{cases} 0 & \text{if child survived} \\ 1 & \text{if not} \end{cases}$$

$$Y = \begin{cases} 0 & \text{no seatbelt} \\ 1 & \text{adult seatbelt} \\ 2 & \text{car seat} \end{cases}$$

Joint distribution table:

| Y \ X | 0 | 1 |
|---|---|---|
| 0 | .38 | .17 |
| 1 | .14 | .02 |

all add up to 1.

$\ldots Y=2) = P(X=1 \wedge Y=2) = p(0,0)$

where $p(x,y)$ ...nt pmf

| $Y$ | | |
|---|---|---|
| 1 | .14 | .02 |
| 2 | .24 | .05 |

$\to P(X=1,\ Y=2) = P(X=1 \wedge Y=2) = p(0,0)$

where $p(x,y)$ is the joint pmf of $X$ and $Y$

for $X, Y$ DRVs, $p(x,y) = P(X=x, Y=y)$ is the joint pmf of $X$ and $Y$ iff

1) $0 \le p(x,y) \le 1 \quad \forall x,y$

2) $\sum_x \sum_y p(x,y) = 1$

$$F(x,y) = P(X \le x, Y \le y) = \sum_{i \le x} \sum_{j \le y} p(i,j)$$

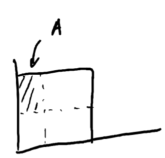
Let $X, Y$ be continuous RVs then $f(x,y)$ is the pdf iff

1) $0 \le f(x,y) \quad \forall x,y$

2) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, \partial x\, \partial y = 1$

$P((X,Y) \in A) = \iint_A f(x,y)\, \partial x\, \partial y \quad$ where $A \subseteq \mathbb{R}^2$

Ex: Let $f(x,y) \begin{cases} 1 & \text{if } 0 < x < 1,\ 0 < y < 1 \\ 0 & \text{o.w.} \end{cases}$




$P(0 \le X \le 1/3,\ 1/2 < Y < 1)$

$$= \int_0^{1/3} \int_{1/2}^{1} 1\, \partial y\, \partial x = \int_0^{1/3} y\Big|_{1/2}^{1} \partial x = \int_0^{1/3} \tfrac{1}{2}\, \partial x = \frac{x}{2}\Big|_0^{1/3} = \frac{1}{6}$$

Ex: let $f(x,y) = \begin{cases} 3x & 0 \le y \le x \le 1 \\ 0 & ow \end{cases}$

$$P(0 \le X \le \tfrac{1}{2},\ \tfrac{1}{4} \le Y) =$$

$$\int_0^{1/2}\int_{1/4}^{x} 3x \, \partial y \, \partial x = \int_0^{1/2} 3xy\Big|_{1/4}^{x} \partial x = \int_0^{1/2} 3x^2 - \tfrac{3}{4}x \, \partial x = x^3 - \tfrac{3}{8}x^2\Big|_0^{1/2} = \frac{1}{8} - \frac{3}{32} = \frac{1}{32}$$

$$= \int_{1/4}^{1}\int_{y}^{1/3} f(x,y)\, \partial x \, \partial y$$

$$F(x,y) = P(X \le x,\ Y \le y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(s,t)\, \partial t \, \partial s$$

$$\left.\begin{aligned} \frac{\partial}{\partial x} F(x) &= f(x) \\ \frac{\partial^2}{\partial y \partial x} F(x,y) &= f(x,y) \end{aligned}\right\} \text{ where these derivatives exist.}$$