

Conditional Probability

Occurrence of one event affects probability of another event.

Ex A, B, X

Ex $P(\text{Roll a 5 or less}) = \frac{1+2+3+4}{36} = \frac{10}{36} = \frac{5}{18}$

Die on table is a 5 $\Rightarrow P(\text{rolling } \leq 5) = 0$
 " " " 3 $\Rightarrow P(\text{rolling } \leq 5) = \frac{1}{3}$
 " " " 1 $\Rightarrow P(\text{rolling } \leq 5) = \frac{2}{3}$

Ex Pick a card

$P(\text{draw H}) = \frac{1}{4} = \frac{13}{52}$

$P(\text{draw H} | \text{drawn red}) = \frac{1}{2} = \frac{13}{26}$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$
 (note that $P(B) > 0$ is required)

↑ ↑
 "given" "conditional event"

Ex: Pick a student at random from a class with the following makeup:

| | So | Jr | Sr | total |
|--------|--------|--------|-------|--------|
| female | 31.44% | 5.71% | 0% | 37.15% |
| male | 40.00% | 17.14% | 5.71% | 62.85% |
| total | 71.44% | 22.85% | 5.71% | 100% |

- 1) What's prob student is soph male? 0.40
- 2) " " male / soph? $40/71.44$
- 3) " " soph / male? $40/62.85$

let A = "student is a sophomore"

B = "student is male"

$$1) P(A \cap B) = 0.4$$

$$2) P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.4}{0.7144}$$

$$3) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.6285}$$

Note $P(A \cap B) = P(B) P(A|B) = P(A) P(B|A)$

Ex: In a class w/ 40% F, 25% of females are Business Majors and 30% of the males are business majors.

- 1) Probability a randomly selected student is a female business major?
- 2) " " " business major?

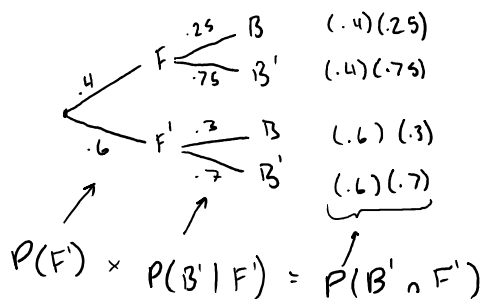
Let F = "student is female"

B = "student is a business major"

$$P(F) = 0.4 \quad P(B|F) = 0.25 \quad P(B|F') = 0.3$$

$$1) P(F \cap B) = P(F) P(B|F) = (0.4)(0.25) = 0.1$$

$$2) P(B) = P(F \cap B) + P(F' \cap B) = 0.1 + P(F') P(B|F') = 0.1 + (0.6)(0.3) = 0.28$$



2.7 is independent events

2.7 is Independent events

events A, B are independent

$$P(A \cap B) = P(B) P(A|B)$$

iff

$$P(A) = P(A|B)$$

iff $P(B) = P(B|A)$

so

$$\text{iff } P(A \cap B) = P(A)P(B)$$

← one to check