# Rec 8/23

Tuesday, August 23, 2016 7:53 AM

## Chapter 1

Combinatorial method.

Sometimes you want to list all possible solutions. This is tedious.

r o y g b i v 7 balls.

pick 3 balls: $\frac{7\cdot6\cdot5}{3\cdot2}$ ← no order $7\cdot6\cdot5$ ← order matters

1) Permutation: distinct arrangement of elements, distinct elements.

# of permutations of n different objects (taking r objects) is:

$$nPr = \frac{n!}{(n-r)!}$$

example: $_7P_3 = \frac{7!}{4!} = 7\cdot6\cdot5 = 210$

2) Combination: distinct elements, not distinct arrangement

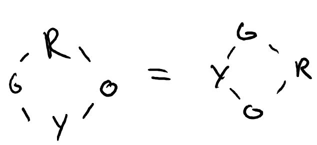
# of combinations of r objects taken from n objects is:

$$nCr = \binom{n}{r} = \frac{nPr}{r!} = \frac{n!}{r!(n-r)!}$$

example: $_7C_3 = \frac{7!}{3!\cdot4!} = 7\cdot5 = 35$

3) You have n distinct objects. # of permutations of them arranged in a circle:

ROYG → permutation: $4!$

but 

, etc. 4 equivalent arrangements.

So it is $\frac{n!}{n} = (n-1)!$

note:

$nP_0 = 1 \qquad nP_n = n! \qquad$ so $\qquad 0! = 1$

$nC_0 = 1 \qquad nC_n = 1 \qquad$ and $\qquad nC_r = nC_{(n-r)}$

Proof: $\quad nC_{(n-r)} = \dfrac{n.}{(n-r)!(n-(n-r))!} = \dfrac{n!}{(n-r)!\,r!} = nC_r$

## Integration techniques (calculus review)

$$I = \int_0^\infty \underbrace{x}_{u}\ \underbrace{e^{-\frac{x}{2}}\,dx}_{dv}$$

integration by parts

$u = x \longrightarrow du = dx$

$dv = e^{-\frac{x}{2}} \longrightarrow v = -2e^{-\frac{x}{2}}$

$$I = u \cdot v - \int v\,du$$

$$= x(-2)\exp\left(-\frac{x}{2}\right)\Big|_{x=0}^{\infty} - \int_0^\infty -2\exp\left(\frac{-x}{2}\right)dx$$

$$= \lim_{x\to\infty} -2x\exp\left(\frac{-x}{2}\right) - \underbrace{0\cdot(-2)\cdot\exp(0)}_{=0} - \int_0^\infty -2\exp\left(\frac{-x}{2}\right)dx$$

$$= \lim_{x\to\infty} \cancel{\frac{-2x}{\exp\left(\frac{x}{2}\right)}} \to 0 - \int_0^\infty -2\exp\left(\frac{-x}{2}\right)dx$$

$$= 2\int_0^\infty \exp\left(\frac{-x}{2}\right)dx \qquad u = \frac{-x}{2} \quad dv = -\frac{1}{2}\,dx \quad x=0 \ \ u=0 \quad x=\infty \ \ u=-\infty$$

$$= -2\cdot(-2)\exp\left(\frac{-x}{2}\right)\Big|_{x=0}^{\infty} \qquad = -2\cdot 2\int_0^{-\infty}\exp(u)\,du = -2\cdot 2\cdot -1$$

$$= 4$$

$I = \int_0^\pi \sin^3 x \cos x\,dx \qquad u = \sin x \quad du = \cos x\,dx \quad \sin(0) = 0 \quad \sin(\pi) = 0$

$$I = \int_0^{\pi} \sin^3 x \cos x \, dx \qquad u = \sin x \quad du = \cos x\,dx \quad \sin(0) = 0 \quad \sin(\pi) = 0$$

$$= \int_0^0 u^3 \, du = 0$$

$$= \int_0^{\pi/2} \sin^3 \cos x \, dx + \int_{\pi/2}^{\pi} \sin^3 x \cos x \, dx \qquad \sin\left(\frac{\pi}{2}\right) = 1$$

$$= \int_0^1 u^3 \, du + \int_1^0 u^3 \, du$$

$$= \int_0^1 u^3 \, du - \int_0^1 u^3 \, du = 0$$

Geometric series

$a \quad ar \quad ar^2 \quad ar^3 \ldots$

$a + ar + ar^2 + ar^3 + \ldots$

$$= \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r} \quad \text{when } |r| < 1$$