Lec 12/2

Friday, December 2, 2016 8:02 AM

Ex: spoze # cigs per day is $Exp(20) \sim X_i$

a) $P(X_i > 20)$

b) $P(\bar{X}_{(4)} > 20)$

c) $P(\bar{X}_{(36)} > 20)$

a) $e^{-x/\lambda} = e^{-1}$

b) $\sum_{i=1}^{4} X_i \sim Gamma(4, 20)$, use calculator

c) $\bar{X}_{(36)} \overset{approx}{\sim} N(20, \frac{400}{36})$, use calculator

$= P(Z > \frac{20-20}{20/6}) = P(Z > 0) = 1/2$

8.5 T distribution

We used CLT when n is large to say that

$\bar{X} \overset{approx}{\sim} N(\mu, \frac{\sigma^2}{n}) \implies Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

don't know this.

σ is generally unknown, instead use sample std. dev S.

Consider the RV $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$ which has 2 sources of variability.

$\Rightarrow$ T is more variable than Z.

$\Rightarrow$ T is not a normal RV (prob)

Thm: If Y, Z are ind. RVs s.t. $Y \sim \chi^2_\nu$ and $Z \sim N(0,1)$

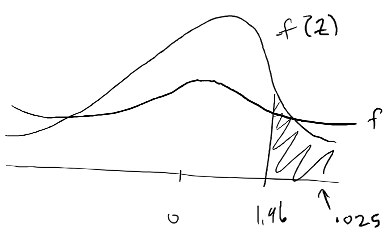
then $T = \frac{Z}{\sqrt{Y/\nu}} \sim T(\nu)$

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

Thm If $\bar{X} \sim N(\mu, \sigma^2/n)$ and $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ and $\bar{X}$ ind S^2

then $T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

$$T = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \Big/ \sqrt{\frac{(n-1)S^2}{\sigma^2 (n-1)}} = \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$$



t dist has fatter tails.

$P(Z > 1.96) = 0.025$

$P(t > 1.96) > 0.025$

$$P(T_5 > 1.96) > P(T_{25} > 1.96) > P(T_{45} > 1.96) > P(Z > 1.96)$$

As $\nu \to \infty$, $t_\nu \xrightarrow{D} N(0,1)$

8.6 F dist

F comes up in regression (14) and ANOVA (15), ratio of 2 variances (11, 13)

Thm: if U, V ind s.t. $U \sim \chi^2_{\nu_1}$ and $V \sim \chi^2_{\nu_2}$

Then $F = \dfrac{U/\nu_1}{V/\nu_2} \sim F_{\nu_1, \nu_2}$

$\left(T \sim t_\nu \Rightarrow T^2 \sim F_{1,\nu} \right)$

$X_i \overset{iid}{\sim} N(\mu_1, \sigma_1^2) \quad Y_j \overset{iid}{\sim} N(\mu_2, \sigma_2^2) \quad X_i, Y_j$ independent.

$$\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1} \qquad \frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

$$\Rightarrow \quad \frac{S_1^2}{\sigma_1^2} \Big/ \frac{S_2^2}{\sigma_2^2} \sim F_{n_1-1, n_2-1}$$