Lec 11/30

Wednesday, November 30, 2016 8:31 AM

Chi Squared Dist:

$X \sim \chi^2_\nu$ $\nu \sim$ degrees of freedom

$X \sim \text{Gamma}\left(\frac{\nu}{2}, 2\right)$

$E(X) = \mu = \nu$

$\text{Var}(X) = \sigma^2 = 2\nu$

$M_X(t) = (1-2t)^{-\nu/2}$

$$f(x;\nu) = \begin{cases} \frac{1}{\Gamma(\frac{\nu}{2}) 2^{\nu/2}} x^{\frac{\nu}{2}-1} e^{-x/2} & \text{if } x > 0 \\ 0 & \text{o.w.} \end{cases}$$

Recall:

1) If $Z \sim N(0,1)$, $Z^2 \sim \chi^2_1$

2) if $X_i \overset{\text{ind}}{\sim} \text{Gamma}(\alpha, \beta)$ then $\sum_{i=1}^n X_i \sim \text{Gamma}(n\alpha, \beta)$

$\Rightarrow$ If $X_i \overset{\text{ind}}{\sim} N(0,1)$ then $Y = \sum_{i=1}^n X_i^2 \sim \text{Gamma}\left(\frac{n}{2}, 2\right)$

$\Leftrightarrow$ $Y \sim \chi^2_n$

Corollary: $X_i \sim \chi^2_{\nu_i} \Rightarrow \sum_{i=1}^n X_i \sim \chi^2_{\sum_{i=1}^n \nu_i}$

Corollary: X_1, X_2 ind, $X_1 \sim \chi^2_{\nu_1}$, $(X_1 + X_2) \sim \chi^2_\nu$ (w. $\nu \geq \nu_1$

then $X_2 \sim \chi^2_{\nu - \nu_1}$

Corollary: $X_1, X_2, \ldots, X_1 \sim \chi^2_{\nu_1}$, $(X_1 + X_2) \sim \chi^2_{\nu}$ (w. $\nu \geq \nu_1$)

then $X_2 \sim \chi^2_{(\nu - \nu_1)}$

thm: Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal pop μ, σ.

Then 1) $\bar{X}$, S^2 independent

2) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$

Proof: $X_i \overset{iid}{\sim} N(\mu, \sigma^2) \Rightarrow \bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ and $Z_i = \frac{X_i - \mu}{\sigma} \sim N(0,1)$

$\Rightarrow \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2_1 \Rightarrow \sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2 \sim \chi^2_n$

but $\sum_{i=1}^{n} \left(\frac{X_i - \mu}{\sigma}\right)^2 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \mu)^2 = \frac{1}{\sigma^2} \sum_{i=1}^{n} (X_i - \bar{X})^2 + n(\bar{X} - \mu)^2$

$\Rightarrow \frac{(n-1)\sum_{i=1}^{n}(X_i - \bar{X})^2}{\sigma^2 (n-1)} + \frac{n(\bar{X} - \mu)^2}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2} + \underbrace{\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}\right)^2}_{\chi^2_1} \sim \chi^2_n$

$\Rightarrow \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$ by Corollary. ■

Define χ^2_α as the number so that $P(X > \chi^2_\alpha) = \alpha$ where $X \sim \chi^2_\nu$

↳ number