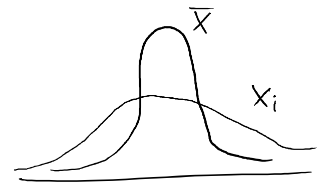
Lec 11/28

Monday, November 28, 2016 7:59 AM

Recall: If $X_1, X_2, \ldots, X_n$ are a random sample from a popolation w/ mean μ and variance σ^2 then

$$E(\bar{X}) = E(X_i) = \mu$$

$$Var(\bar{X}) = \frac{Var(X_i)}{n} = \frac{\sigma^2}{n}$$



If $X_i \overset{iid}{\sim} N(\mu, \sigma^2)$ then $\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$

for <u>any</u> Dist.

As $n \to \infty$, $Var(\bar{X}) = \frac{\sigma^2}{n} \to 0$

As $n \to \infty$ $\bar{X}_n \xrightarrow{P} \mu$

Let $\varepsilon > 0$. Then $\lim_{n\to\infty} P(|\bar{X}_n - \mu| > \varepsilon)$ (chebyshev's Ineq: $P(|X-\mu| > k\sigma) \le \frac{1}{k^2}$)

$$= \lim_{n\to\infty}\left(P(|\bar{X}_n - \mu| > k\sigma) \le \frac{\sigma^2}{\varepsilon^2 n}\right) = 0$$

$\to 0.$

let $\varepsilon = \frac{k\sigma}{\sqrt{n}}$

$\Rightarrow k = \frac{\varepsilon\sqrt{n}}{\sigma}$

$k^2 = \frac{\varepsilon^2 n}{\sigma^2}$

$\bar{X}$ is consistent for μ (as $n \to \infty$, it converges in probability to the parameter).

For large n, $\bar{X} \doteq N(\mu, \frac{\sigma^2}{n})$ (Central Limit Theorem). (p 234-235).

as $n \to \infty$, $\bar{X} \rightsquigarrow N(\mu, \frac{\sigma^2}{n})$

for any ε, $\int_{\mathbb{R}} |f_{\bar{X}_n}(x) - f_N(x)|\, dx < \varepsilon$ for $n >$ some M.

X_i ind ...

$$X_i \overset{ind}{\sim} \text{Gamma}(\alpha, \beta)$$

$$\sum_{i=1}^{n} X_i \sim \text{Gamma}(n\alpha, \beta)$$

$$\bar{X} \sim \text{something}$$