

# Lec 11/21

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Common stats:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$$

$$\tilde{X} = \text{median}$$

$$S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$$

$$X_1 \leq \dots \leq X_n \quad \text{order statistics.}$$

8.2: Sampling distribution of the sample mean.

$\bar{X} \leftarrow \text{RV.}$   $\rightarrow$  Dist. of a statistic.

if  $X_i \stackrel{iid}{\sim} N(\mu, \sigma^2)$

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

In general, if  $X_1, \dots, X_n$  are a random sample <sup>iid</sup> from a population with mean  $\mu$  and variance  $\sigma^2$ ,

then  $E(\bar{X}) = \mu = E(X_i)$

and  $\text{Var}(\bar{X}) = \frac{\sigma^2}{n} = \frac{\text{Var}(X_i)}{n}$

$$E(\bar{X}) = E\left(\frac{1}{n} \sum X_i\right) \stackrel{\leftarrow \text{id dist.}}{=} \frac{1}{n} \cdot n \mu = \mu.$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{n} \sum X_i\right) = \frac{1}{n^2} \text{Var}\left(\sum X_i\right) = \frac{1}{n^2} \sum \text{Var}(X_i) = \frac{1}{n^2} \cdot n \cdot \sigma^2 = \frac{\sigma^2}{n}$$

$\uparrow$  iid, id dist.

as  $n \rightarrow \infty$ ,  $\text{Var}(\bar{X}) = \frac{\sigma^2}{n} \rightarrow 0$