Lec 11/16

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Joint dist. of u and v:

$$f(u,v) = \left(\frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{u-0}{\sqrt{2}}\right)^2} \right)\left(\frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{v-0}{\sqrt{2}}\right)^2} \right)$$

where X, Y iid $N(0,1)$, $U = X+Y$ $V = X-Y$ U, V ind

$U \sim N(0, \overset{\sigma^2}{2})$ $V \sim N(0, \overset{\sigma^2}{2})$ U, V iid $N(0,2)$

$X_i \sim N(\mu_i, \sigma_i^2)$ $a_i\, b_i \in \mathbb{R}, a_i \neq 0$

$$Y = \sum_{i=1}^{n} (a_i X_i + b_i) \sim N\left(\sum_{i=1}^{n} (a_i \mu_i + b_i), \sum_{i=1}^{n} (a_i^2 \sigma_i^2) \right)$$

7.5 MGF technique

relies on the fact that the MGF uniquely determines the distribution. MGF may not exist, if so this cannot be used.

shines when trying to find the dist of a linear comb. of RVs for which MGF is known.

Thm: If $X_1, \dots, X_n$ are ind. RVs and $Y = X_1 + \dots + X_n$ and $M_{X_i}(t)$ exists,

then $M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$

Pf: $M_Y(t) = E(e^{Yt}) = \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} e^{ty} f(x_1, \dots, x_n)\, dx_1 \cdots dx_n$

$$= \int_{\mathbb{R}} \cdots \int_{\mathbb{R}} e^{t(x_1 + \dots + x_n)} f(x_1) \cdots f(x_n)\, dx_1 \cdots dx_n$$

$$= \int e^{tx_1} f(x_1)\, dx_1 \cdot \dots \cdot \int_{\mathbb{R}} e^{tx_n} f(x_n)\, dx_n$$

$$= \int_{\mathbb{R}} e^{tx_1} f(x_1)\,\partial x_1 \cdot \dots \cdot \int_{\mathbb{R}} e^{tx_n} f(x_n)\,\partial x_n$$

$$= E\left(e^{tX_1}\right) \cdot \dots \cdot E\left(e^{tX_n}\right)$$

$$= \prod_{i=1}^{n} M_{X_i}(t)$$

Ex: X_i ind $P_o(\lambda_i)$, $Y = \Sigma X_i$, $M_{X_i}(t) = e^{\lambda_i (e^t - 1)}$

$$M_Y(t) = \prod_{i=1}^{n} M_{X_i}(t)$$

$$= e^{(\Sigma \lambda_i)(e^t - 1)}$$

So $Y \sim P_o(\Sigma \lambda_i)$