Lec 10/7

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If X_is are ind:

$$Var(Y) = Var\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 \, Var(X_i)$$

where $Y = a_1X_1 + a_2X_2 + \dots + a_nX_n$

Sample mean: assume X_is are identically distributed.

$$E(\bar{X}) = E\left(\sum_{i=1}^{n} \frac{1}{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{n}\sum_{i=1}^{n} \mu = \mu$$

$$E(\bar{X}) = E(X_i) = \mu$$

Sampling distribution

X_i : height (in) of i^{th} person (60 people)

samples of size 2: $\binom{60}{2} = 1770$

$\overline{X_i}$ = avg ht of 2 ppl in sample.

$$\frac{\sum_{i=1}^{1770} \overline{X_i}}{1770} = E(X_i) = \mu = \frac{\sum_{i=1}^{60} X_i}{60}$$

$$Var(\bar{X}) = Var\left(\sum_{i=1}^{n} \frac{1}{n} X_i\right) \overset{\text{independent}}{=} \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i) \overset{\text{identically dist.}}{=} \frac{Var(X_i)}{n}$$

$Var(\bar{X}) < Var(X_i)$ which is why people take samples for $n > 2$

as $X \uparrow$ $Var(\bar{X}) \downarrow$

Ex: Let X, Y, Z be RVs. Let $W = 2X - Y + Z$

$$E(W) = 2E(X) - E(Y) + E(Z)$$

$$Var(W) = Var(2X - Y + Z) =$$

$$4Var(X) + Var(Y) + Var(Z) + 2(-2Cov(X,Y) + 2Cov(X,Z) - Cov(Y,Z))$$

Let $X_1, X_2, \ldots, X_n$ be RVs, $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ constants.

Define $Y_1 = \sum_{i=1}^{n} a_i X_i$

$Y_2 = \sum_{i=1}^{n} b_i X_i$

$$Cov(Y_1, Y_2) = Cov\left(\sum_{i=1}^{n} a_i X_i, \sum_{i=1}^{n} b_i X_i\right)$$

$$= \sum_{i=1}^{n} a_i b_i Var(X_i) + \sum_{i<j}^{n}\sum (a_i b_j + a_j b_i) Cov(X_i, X_j)$$

$$\left.\begin{array}{l} Cov(\bar{X}, S^2) \\ S^2 = \dfrac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1} \end{array}\right\}$$

useful for this

(sample variance).

Ex: X, Y, Z RVs,

$$W_1 = 2X - Y + Z$$

$$W_2 = -4X + 3Y - Z$$

$$Cov(W_1, W_2) = Cov(2X - Y + Z, -4X + 3Y - Z)$$

$$= Cov(2X, -4X) + Cov(2X, 3Y) + Cov(2X, -Z)$$
$$+ Cov(-Y, -4X) + Cov(-Y, 3Y) + Cov(-Y, -Z)$$
$$+ Cov(Z, -4X) + Cov(Z, 3Y) + Cov(Z, -Z)$$

$$= -8\,Var(X) - 3\,Var(Y) - Var(Z) + 10\,Cov(X,Y) - 6\,Cov(X,Z) + 4\,Cov(Y,Z)$$