Lec 10/5

Wednesday, October 5, 2016 8:13 AM

$$\text{Cov}(aX+c,\ bX+d) = ab\,\text{Cov}(X,Y)$$

Covariance of n vars:

NOT:

$$\text{Cov}(X,Y,Z) = E[(X-\mu_X)(Y-\mu_Y)(Z-\mu_Z)]$$

so

$$\text{Cov}(X,X,X) \neq \text{Var}(X)$$

$X_1, X_2, \ldots, X_n$ are RVs

$\text{Cov}(X_i, X_j)$ makes sense $\rightarrow = \text{Cov}(X_j, X_i)$

for $i \neq j$, covariance

$i = j$, variance

Variance-Covariance matrix: $\not\equiv$

matrix whos ith row, jth column entry is $\text{Cov}(X_i, X_j)$

$$\begin{bmatrix} \text{Var}(X_1) & \text{Cov}(X_1, X_2) & & \vdots \\ \text{Cov}(X_2, X_1) & \text{Var}(X_2) & \ddots & \\ & & & \text{Cov}(X_n, X_{n-1}) \\ \cdots & & \text{Cov}(X_{n-1}, X_n) & \text{Var}(X_n) \end{bmatrix}$$

4.7: Moments of Linear Combinations of RVs

$$\sum_{i=1}^{n} a_i X_i$$

X_i are random vars, $a_i = \frac{1}{n}$ for all i: $\sum_{i=1}^{n} \frac{1}{n} X_i = \bar{X}$ ("X bar" - sample mean)

μ_X ← (constant) parameter

$\bar{X}$ ← statistic (RVs)

Call $\sum_{i=1}^{n} a_i X_i = Y$ where X_i are RVs, a_i are constants

Y is a RV.

$$E(Y) = E\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i E(X_i)$$

(proof on pg. 135)

$$Var(Y) = Var\left(\sum_{i=1}^{n} a_i X_i\right) = \underbrace{\sum_{i=1}^{n} a_i^2 \, Var(X_i) + 2 \sum_{i<j}^{n} \sum a_i a_j \, Cov(X_i, X_j)}_{\text{adding every entry in Variance-Covariance matrix}}$$

$$= E\left[\left(\sum_{i=1}^{n} a_i X_i\right) - \left(\sum_{i=1}^{n} a_i E(X_i)\right)\right]$$

* When there is independence: $Var(Y) = \sum_{i=1}^{n} a_i^2 \, Var(X_i)$