Lec 10/24

Monday, October 24, 2016 8:20 AM

6.2: Uniform distribution

$X \sim \text{Unif}(a, b)$

pdf: $X \in (a, b)$ or $[a, b]$

$$f(x; a, b) = \begin{cases} \frac{1}{b-a} & \text{if } x \in (a,b) \\ 0 & \text{o.w.} \end{cases}$$

$E(X) = \frac{a+b}{2}$ → true for any symmetric distribution.

$$P(\alpha < X < \beta) = \frac{\beta - \alpha}{b-a}$$

$$P(X < \beta) = \frac{\beta - a}{b-a}$$

CDF: $F(x; a, b) = \frac{x-a}{b-a}$

$$Var(X) = \int_a^b \frac{x^2}{b-a}\,dx - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{1}{3(b-a)}(b^3 - a^3) - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b+a}{3} - \frac{a^2 + 2ab + b^2}{4}$$

$$= \frac{(b-a)^2}{12}$$ = Moment of inertia of a bar about the CM

Ex: X = time after 10:30 of the arrival of 1st student to my office hours.

$X \sim \text{Unif}(0, 60)$

a) $P(X > 30) = \frac{30-0}{60-0} = \frac{1}{2}$

6.3: Gamma, χ^2, Exponential Dists.

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\,dt \qquad \alpha > 0.$$

It can be shown (IBP) that

$$\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1) \qquad \alpha > 0$$

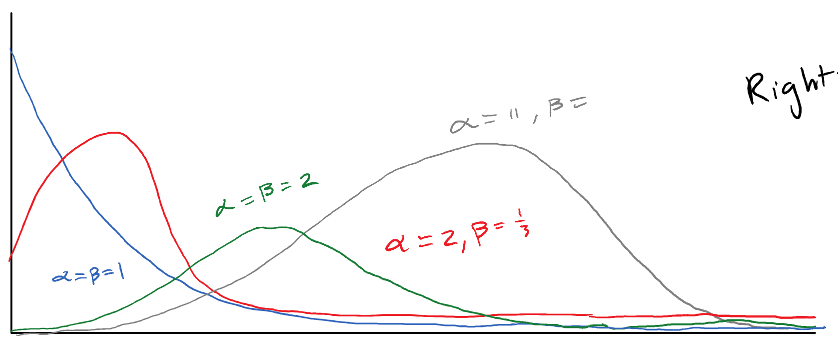
$\Rightarrow$ if $n \in \mathbb{N}$, $\Gamma(n) = (n-1)!$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

$X \sim \text{Gamma}(\alpha, \beta)$ iff

$$f(x; \alpha, \beta) = \begin{cases} \dfrac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha)\beta^{\alpha}} & \text{if } x > 0 \\ 0 & \text{o.w.} \end{cases}$$

$\alpha > 0$
$\beta > 0$



Right-skewed

$$E(X) = \alpha\beta$$

$$Var(X) = \alpha\beta^2$$

$$M_X(t) = (1 - \beta t)^{-\alpha}$$

The gamma has special cases that arise in many places in statistics.

If we let $\alpha = 1$, $\Gamma(1) = 1$, $f(x; \beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & x > 0 \\ 0 & \text{o.w} \end{cases}$

this is the exponential distribution.

$$X \sim Exp(\lambda)$$

$$E(X) = \lambda$$

$$Var(X) = \lambda^2$$

$$M_X(t) = \frac{1}{1-\lambda t}$$