Lec 10/19

Wednesday, October 19, 2016 8:12 AM

Hypergeometric RV.

X = # successes in n trials.

- Ind
- Const prob of success

} ← assumptions

$X \sim Bin(p)$

If its a random sample from an 'infinite' population, these are good.

or from a pop. much larger than n (sample size).

But what if we sample from a finite population that is not significantly larger than the sample size:

Binomial does not apply.

Hypergeometric

Ex: Suppose randomly selecting 10 ppl from 100 where the 100 consist of 65 dems & 35 reps. probability we get x dems in the 10.

$$P(X=x) = \frac{\binom{65}{x}\binom{35}{10-x}}{\binom{100}{10}}$$

Generalization. Consider a pop. of size N comprised of 2 groups:

M have the characteristic (aka are successes), so $N-M$ do not (aka are failures).

Select n elements at random, and let X = # successes in the n elems.

Then $X \sim HypGeom(n, N, M)$

and $p(x;\ n, N, M) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ for $x = 1, \ldots, n$

and $p(x;\ n, N, M) = \dfrac{\cdots}{\binom{N}{n}}$ for $x = 1, \ldots, n$

$$E(X) = n\left(\frac{M}{N}\right)$$

$$Var(X) = \left(\frac{N-n}{N-1}\right) n\left(\frac{M}{N}\right)\left(1 - \frac{M}{N}\right) \qquad np(1-p)$$

finite population correction factor: fpcf

Note: binomial is fine as long as $n < (0.05)N$

5.7: Poisson Distribution

Consider X = # OSU students w/ a medical emergency on a given day.

$n = 50{,}000 \qquad p = 0.0001$

$$P(X = 12) = \binom{50000}{12}(0.0001)^{12}(1-0.0001)^{50000-12}$$

Consider the Binomial dist. where $n \to \infty$, $p \to 0$, but np remains const.

Let $\lambda = np \Rightarrow p = \frac{\lambda}{n}$

$X \sim Bin(n, p)$

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \frac{n(n-1)(n-2)\ldots(n-x+1)}{x!}\left(\frac{\lambda}{n}\right)^x\left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)}{x!}\ \lambda^x \left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}$$

$$\left[\left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}}\right]^{-\lambda}$$

$$\lim_{n\to\infty}\left(1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{x-1}{n}\right)\right) = 1$$

$$\lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{-x} = 1$$

$$\lim_{n\to\infty} \left(1-\frac{\lambda}{n}\right)^{-\frac{n}{\lambda}} = e$$

$$P(X=x) \to \frac{e^{-\lambda}\lambda^x}{x!}$$

$X \sim \text{Poisson}(\lambda)$ means

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \text{for } x = 0, 1, \ldots$$

X = # events/successes in a time/space frame.

$$M_X(t) = e^{\lambda(e^t - 1)}$$

$$E(X) = \lambda$$

$$Var(X) = \lambda$$

Poisson can be used to approximate binomial if $n \geq 20$ and $p \leq 0.05$