Lec 10/12

Wednesday, October 12, 2016 8:08 AM

Binomial dist:

$$p(x;\theta;n) = \binom{n}{x}\theta^{x}(1-\theta)^{n-x}$$

Ex: 20 questions on exam, 5 choices.

a) probability student passes:

$$P(X \geq 12) = P(X=12) + P(X=13) + \dots + P(X=20)$$

$$= 1 - P(X < 12) = 1 - \text{binomcdf}(20, 0.2, 11)$$

b) Probability 0 correct

$$P(X=0) = \binom{20}{0}(.2)^0(.8)^{20} \approx 0.0115$$

c) Expected value: $E(X) = n\theta = 20 \cdot 0.2 = 4.$

Claim: $E(X) = n\theta$

Proof: $E(X) = \sum_{x=0}^{n} x\, p(n, \theta, x)$

$$= \sum_{x=1}^{n} x \binom{n}{x}\theta^{x}(1-\theta)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{x\, n!}{x!(n-x)!}\theta^{x}(1-\theta)^{n-x}$$

$$= n\theta \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-1-(x-1))!}\theta^{x-1}(1-\theta)^{n-1-(x-1)}$$

$$= n\theta \sum_{y=0}^{m} \frac{m!}{y!(m-y)!}\theta^{y}(1-\theta)^{m-y}$$

..

$$= n\theta \sum_{y=0}^{m} \text{binompdf}(m, \theta, y)$$

$$= n\theta \qquad \blacksquare$$

$$E(X^2 - X) = E(X^2) - E(X)$$

$$\parallel$$

$$E(X(X-1))$$

$$\parallel$$

$$\vdots$$

something (using same technique as above)

Proof 2: $E(X) = E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} \theta = n\theta$

$\mathrm{Var}(X) = \mathrm{Var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \mathrm{Var}(X_i) = \sum_{i=1}^{n} \theta(1-\theta) = n\theta(1-\theta)$

Moment generating function:

$$M_X(t) = E(e^{tX}) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

$$= \sum_{x=0}^{n} \binom{n}{x} (e^t \theta)^x (1-\theta)^{n-x}$$

$$= (\theta e^t + (1-\theta))^n \qquad \text{(binomial thm).}$$

$$= (1 + \theta(e^t - 1))^n$$

If $X \sim Bin(n, \theta)$ and $Y = \frac{X}{n}$ is the proportion of successes.

$E(Y) = \theta$

$Var(Y) = \frac{1}{n}\theta(1-\theta)$

Let $c > 0$. $P(-c < Y - \theta < c)$

$= P(\theta - c < Y < \theta + c) \to 1$ as $n \to \infty$

Convergence in probability

$\lim_{n \to \infty} P(|Y - \theta| < \varepsilon) = 1$

$\lim_{n \to \infty} P(|Y - \theta| > \varepsilon) = 0$

Y has n+1 possibilities.

as $n \to \infty$, Y acts like a CRV