9/4

Wednesday, September 4, 2019 15:01

L - k-component link

$$H_1(S^3 \setminus L) = \underbrace{\mathbb{Z} \oplus \cdots \oplus \mathbb{Z}}_{k}$$

* Alexander Duality

$$H^{n-1-k}(X) = H_k(S^n - X)$$

$$\begin{array}{c} \tilde{X} \\ \downarrow \\ X \end{array}$$

$$\pi_1(X) \twoheadrightarrow H$$

* Hurewicz Thm

$$\pi_L = \pi_1(S^3 \setminus L)$$

$$\pi_L / [\pi_L, \pi_L] \cong H_1(S^3 \setminus L) = \mathbb{Z}^k$$

Basic Examples

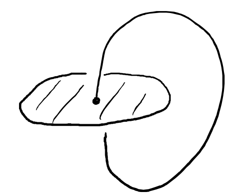
$$S^3 = \{(z, w) : |z|^2 + |w|^2 = 1\} \subseteq \mathbb{C}^2 = \mathbb{R}^4$$

$$C_1 = S^3 \cap (\mathbb{C} \times 0) = \{(z, 0) : |z|^2 = 1\}$$

$$\mathcal{D}_1 = S^3 \cap (\mathbb{C} \times \mathbb{R}_{\geq 0}) = \{ (z, \sqrt{1-|z|^2}) \ : \ |z| < 1 \} \cong D^2.$$

$$\mathcal{C}_2 = S^3 \cap (0 \times \mathbb{C}) \ , \quad \mathcal{D}_2 = S^3 \cap (\mathbb{R}_{\geq 0} \times \mathbb{C})$$

$$\mathcal{C}_1 \cap \mathcal{D}_2 = \{ (1,0) \}$$



$$H = \mathcal{C}_1 \sqcup \mathcal{C}_2 \subset S^3$$

= Hopf link

$$S^3 - \mathcal{C}_1 = S^3 \cap \left(\mathbb{C} \times (\mathbb{C} - 0) \right)$$

$|w| > 0$

$$(z, \xi) \longmapsto (z, \xi \sqrt{1-|z|^2})$$

$$\mathring{D}^2 \times S^1 \xrightarrow{\approx} S^3 - \mathcal{C}_1$$

$$\pi_1 (S^3 \setminus \mathcal{C}_1)$$

$$= \pi_1 (\mathring{D}^2 \times S^1)$$

$$= \pi_1 (S^1) = \mathbb{Z}$$

$$S^3 \setminus H = \{ (z, w) : |z| > 0, \ |w| > 0 \}$$

$$(0,1) \times S^1 \times S^1 \xrightarrow{\approx} S^3 \setminus H$$

$$(r, \xi, \zeta) \longmapsto (r\xi, \sqrt{1-r^2}\,\zeta)$$

$$\pi_1 (S^3 \setminus H) = \pi_1 ((0,1) \times S^1 \times S^1)$$

$$= \pi (S^1 \times S^1)$$

$$= \mathbb{Z} \oplus \mathbb{Z}$$

has $\pi_1 = F_2 = \mathbb{Z} * \mathbb{Z}$

$$p : S^3 \longrightarrow S^2 = \{ (u, x) \in \mathbb{C} \times \mathbb{R} : |u|^2 + x^2 = 1 \}$$

$$P(z, w) = (2z\bar{w}, \ |z|^2 - |w|^2)$$

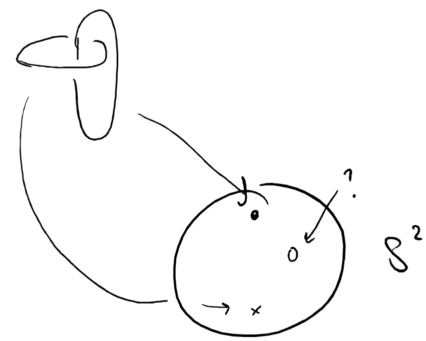
$\forall a \in S^2, \quad p^{-1}(a) = \text{circle}$

$$S^1 \hookrightarrow S^3$$

$$\begin{array}{ccc} S^1 & \hookrightarrow & S^3 \\ & & \downarrow \\ & & S^2 \end{array}$$

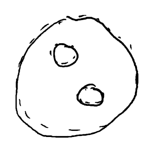
$$C_1 = p^{-1}(0,1)$$

$$C_2 = p^{-1}(0,-1)$$



$$p^{-1}(\{a,b,c\}) = 3\text{- comp link} = \mathcal{L}$$

* find in link table

* $S^3 \setminus \mathcal{L} =$  $\times S^1$

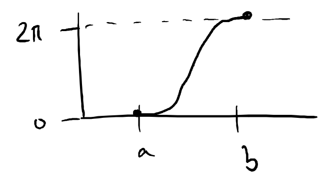
$$\pi_{\mathcal{L}} = F_2 \oplus \mathbb{Z}$$

(Dehn) Twists

$$D_1^2 \subset \mathbb{R}^2$$

$$\rho : D_1^2 \times [a,b] \longrightarrow D_1^2 \times [a,b]$$

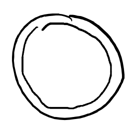
$$\theta : [a,b] \longrightarrow \mathbb{R}$$

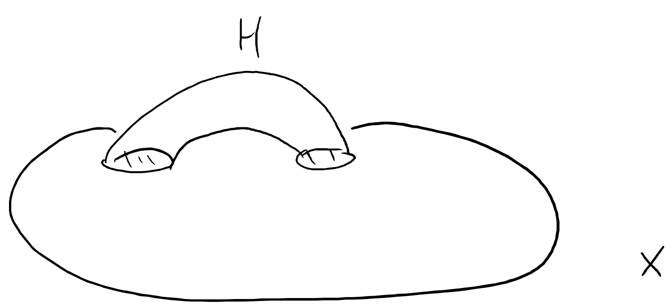


$$R(\varphi) = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

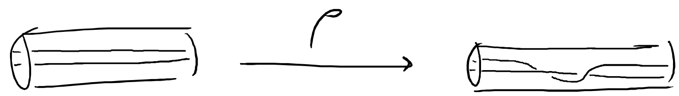
$$\rho(\vec{v}, x) = \left(R(\theta(x))\, \vec{v},\ x \right)$$

$$\rho^{-1}(\vec{v}, x) = \left(R(\theta(x))^{-1} \vec{v},\ x \right)$$ — homeomorphism

* restrict to  $\times [a,b]$

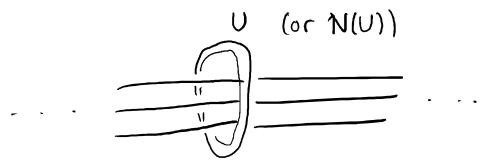


$$\begin{tikzcd} X \cup H \arrow[r] \arrow[r, bend left, "id_X"] \arrow[r, bend right, "\rho"'] & X \cup H \end{tikzcd}$$

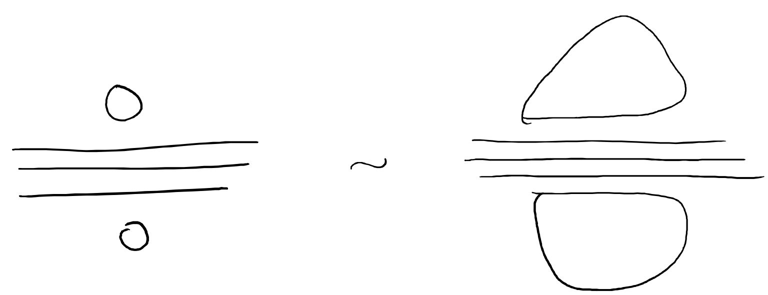


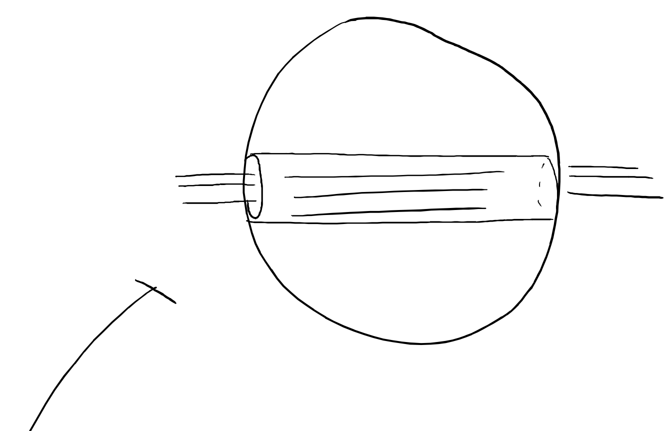
Suppose $\mathcal{L} \subset S^3$ PL-link

Certain unknot component U



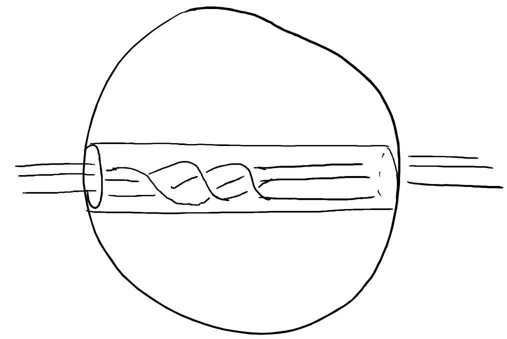
$S^3 \setminus U$





$S^3 \setminus N(U) \circlearrowleft \rho$

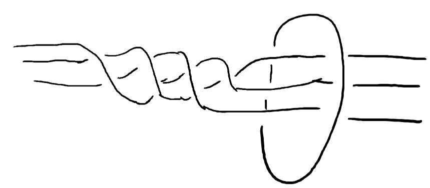
$$S^3 \setminus N(U) \circlearrowleft \rho$$



Obtain homeomorphism

$$S^3 \setminus \mathcal{L} \longrightarrow S^3 \setminus \mathcal{L}'$$

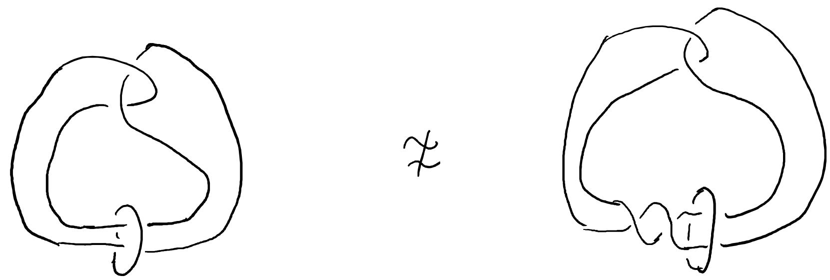
where $\mathcal{L}'$ looks like



This yields homeom

$$S^3 \setminus \mathcal{L} \approx S^3 \setminus \mathcal{L}'$$

for inequivalent links



Whitehead Link L_1 $\qquad\not\approx\qquad$ something else L_2

but $S^3 \setminus L_1 \approx S^3 \setminus L_2$

if $S^3 \setminus L_1 \xrightarrow{\approx} S^3 \setminus L_2$

preserves meridians, $L_1 \approx L_2$.

<u>Thm</u> [Gordan, Luecke 1989]

Suppose $K, K' \hookrightarrow S^3$ are tame knots

and $S^3 \setminus K \underset{\text{homeom}}{\approx} S^3 \setminus K'$.

Then K' is equiv to $K, \bar{K}, K^{-1}, \bar{K}^{-1}$.

<u>Knot Projections</u>:

$$K \hookrightarrow \mathbb{R}^3$$

$$P : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$$

bad things: