9/27

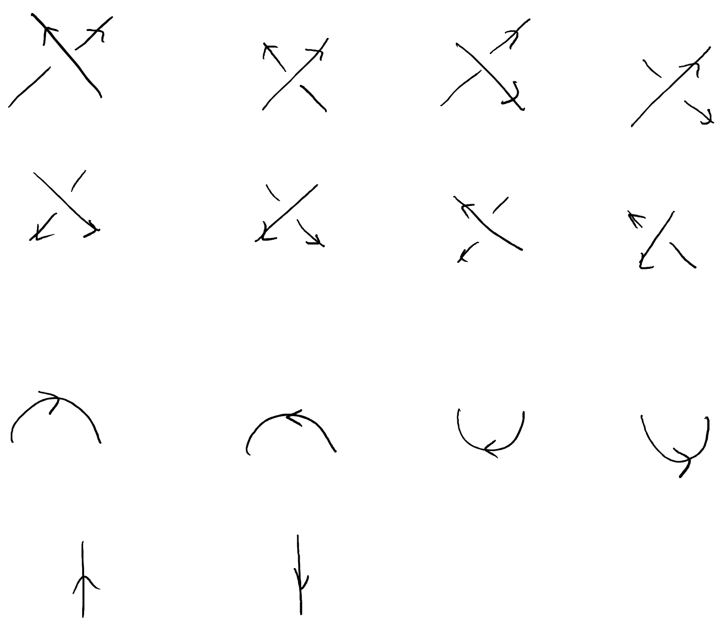
Friday, September 27, 2019 14:59

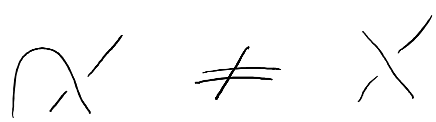
$$(x_1, x_2, x_3) \stackrel{p}{\longmapsto} (x_1, x_2) \stackrel{q}{\longmapsto} x_1$$

PL-link $q \circ p$ generic: all segments map injectively to $\mathbb{R}$.

Local pictures (up to hight preserving isotopies)

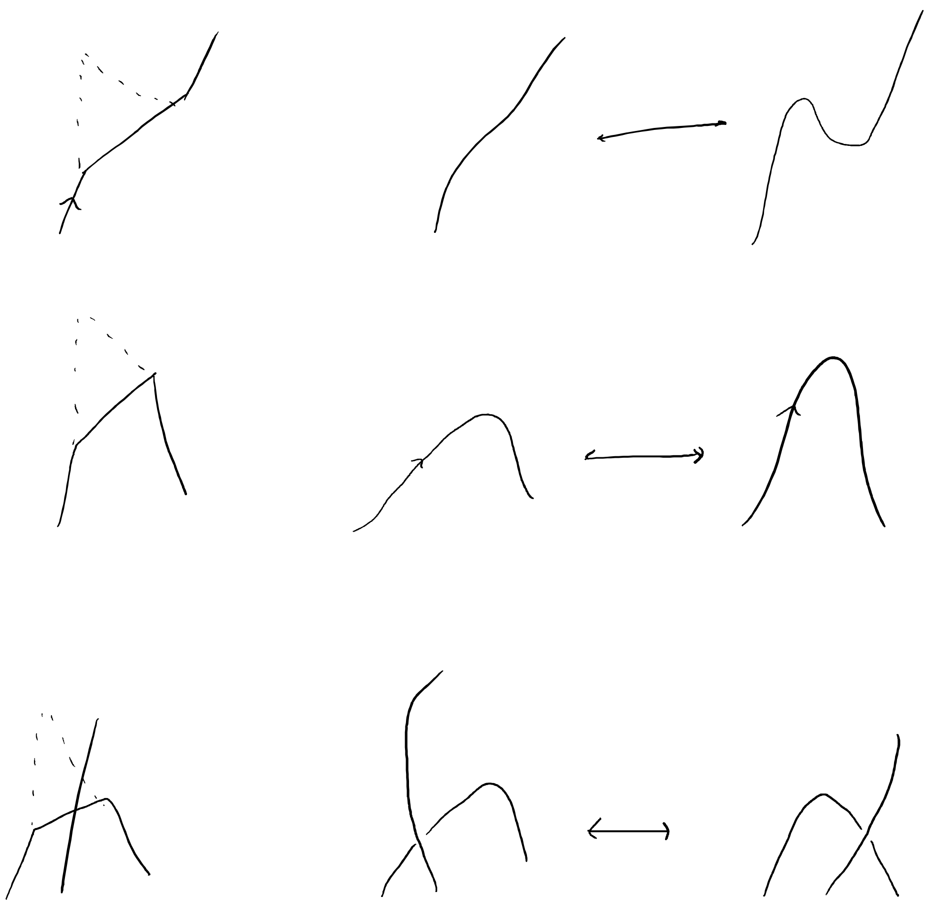
(after smoothening)





$$\hat{L} = \hat{L}_0 \longrightarrow \hat{L}_1 \longrightarrow \cdots \longrightarrow \hat{L}_N = \hat{L}'$$

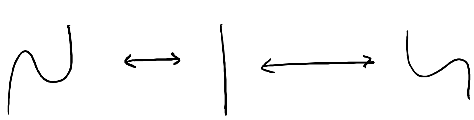
$L \sim L'$ (0) – (4) elementary Δ-moves

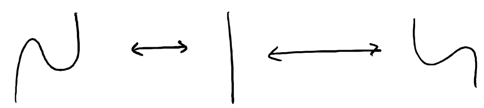


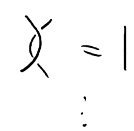
(HI) = height-preserving isotopy

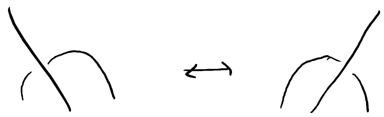
(vRI), (vRII), (vRIII) = vertical Reidemeister moves

(E) =



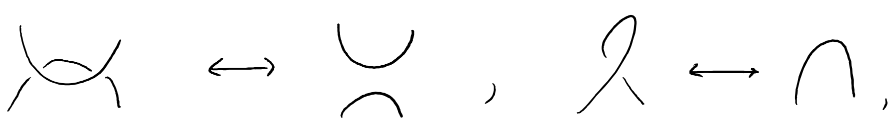
$=$ 

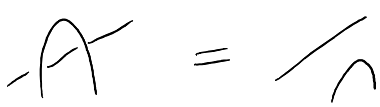


(C) =  & other crossing

(V) = Vertical unobstructed movements of crossings & extrema.

THEOREM If L and L' are equivalent with pq-generic projections $\hat{L}$ and $\hat{L}'$, there is a sequence of above moves connecting $\hat{L}$ & $\hat{L}'$.

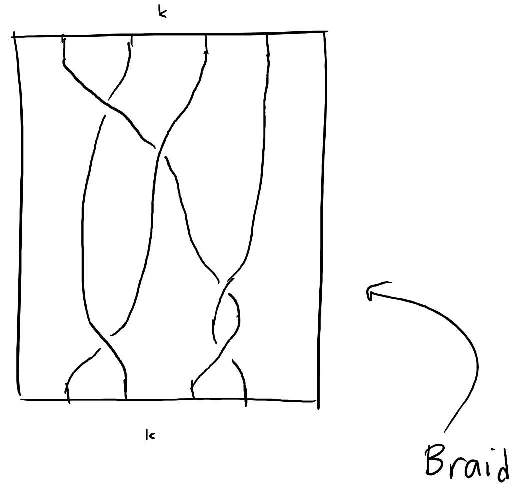
Ex  etc



Braids

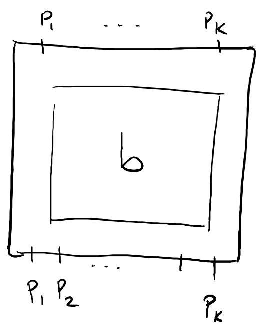
$\hat{L}$ rectangle $[l_1, l_2] \times [h_1, h_2]$

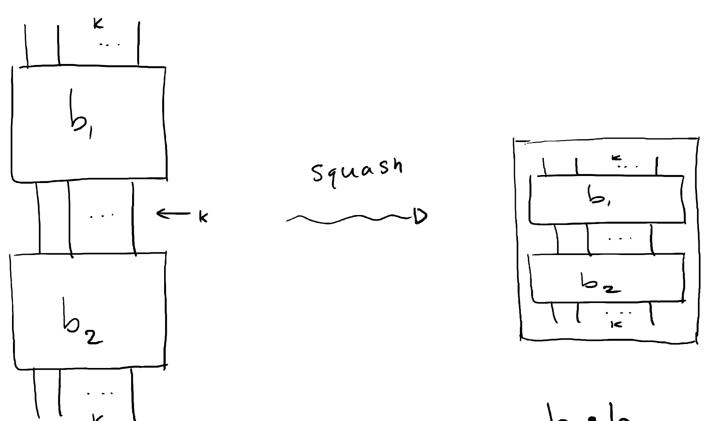
* no extrema



* no extrema in rectangle
* $\hat{L} \cap \partial R$ only in top & bottom

↪ standard rectangle $[0,1]^2$





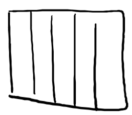
$b_1 \cdot b_2$

$b \sim b'$ equiv. as before but fixed boundary points.

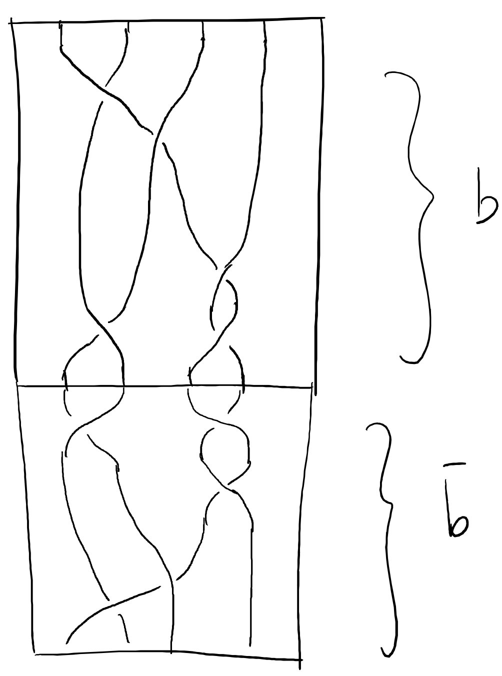
* $b_1 \sim b_1'$; $b_2 \sim b_2'$

  $\Rightarrow b_1 \circ b_2 \sim b_1' \circ b_2'$

* $(b_1 \circ b_2) \circ b_3 \sim b_1 \circ (b_2 \circ b_3)$

* $e =$  , $e \circ b \sim b \circ e \sim b$

* let $\bar{b}$ mirror b along horizontal line



$\bar{b} \circ b \sim b \circ \bar{b} \sim e$

$[b]$ = equivalence classes

$$[b_1]\cdot[b_2] = [b_1 \cdot b_2]$$

$\rightsquigarrow$ group, denoted B_k, the braid gp on k strands.

$$B_2 = \mathbb{Z}$$

$$B_3 = \widetilde{PSL(2,\mathbb{Z})}$$

$$B_k \twoheadrightarrow S_k \quad \text{gp hom.}$$

$$b \longmapsto \sigma$$

$$P_k \hookrightarrow B_k \twoheadrightarrow S_k$$

$\uparrow$

Pure braid group (1 goes to 1, etc)

$$P_k \lhd B_k$$

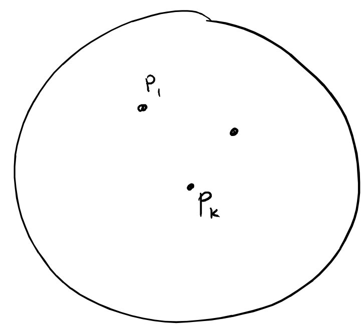
$$B_k = \pi_1\left(\mathrm{Config}_k(\mathbb{R}^2)/S_k\right)$$

$$\mathrm{Config}_k(\mathbb{R}^2) = (\underbrace{\mathbb{R}^2 \times \mathbb{R}^2 \times \cdots \times \mathbb{R}^2}_{k}) \setminus \Delta$$

fat diagonal

$$= \{(x_1, \dots, x_k) : x_i = x_j \text{ for some } i \neq j\}$$

$$B_k = \Gamma^+(S^2 \setminus \{p_1, \dots, p_k\})$$



Σ - oriented surface

$\varphi : \Sigma \longrightarrow \Sigma$ orientation-preserving self-homeomorphisms

$\rightsquigarrow$ form $\mathrm{Homeom}^+(\Sigma)$

$$\cup$$

$\mathrm{Homeom}^+(\Sigma)^\circ \longleftarrow$ homeom isotopic to id_Σ.

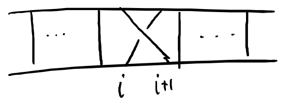
$$\Gamma^+(\Sigma) = \frac{\mathrm{Homeom}^+(\Sigma)}{\mathrm{Homeom}^+(\Sigma)^\circ}$$

Exercise:

$b \sim b' \iff b$ equiv b' using only (vRII), (vRIII), (V)

$\Rightarrow$ presentation of B_k.

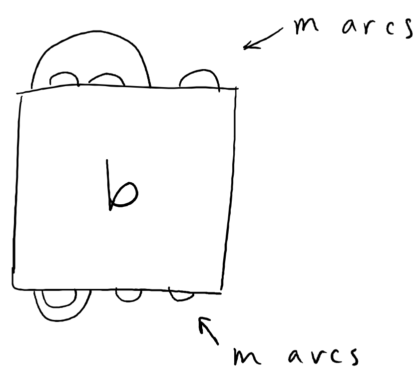
$$B_k = \left\langle \sigma_1, \dots, \sigma_{k-1} \;\middle|\; \begin{array}{l} \sigma_i \sigma_j = \sigma_j \sigma_i \quad \text{if } |i-j| \geq 2 \\ \sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1} \end{array} \right\rangle$$

$\sigma_i =$ 

$L =$ link diagram w/ m maxima.

(m minima too)

Then

$L \approx$ 

for any arc config, $\exists b$.

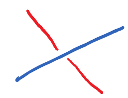
Pf pull up maxima, pull down minima.

Def Minimal number of maxima of any braid presentation of L is called $b(L)$,

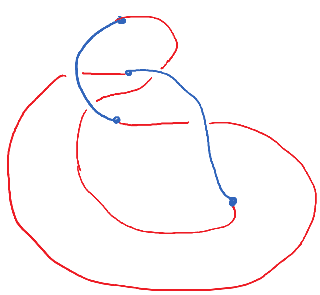
the bridge number.

Original definition of b(L)...

Bridges of Link-diagram $\hat{L}$ of L

- Divide L into blue & red segments s.t. at each crossing in $\hat{L}$ looks like 

b(L) is minimal number of red segments (or blue segments)



b() = 2

had to introduce another crossing to minimize...

.. blue arcs above, points in page, red arcs below

⇝ the two defns agree.