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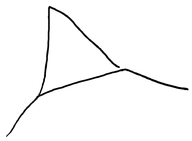
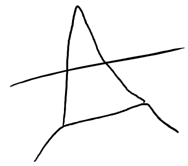
Wednesday, September 25, 2019 14:56

Elementary Planar Δ-moves:

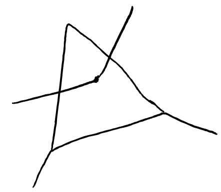
(0)



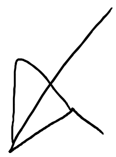
(1)



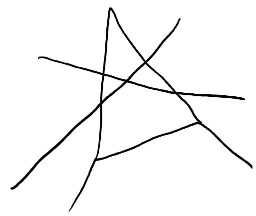
(2)



(3)



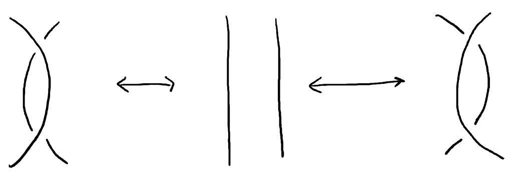
(4)



+ obvious variations.

(R0) = Planar isotopies (of $\mathbb{R}^2$).

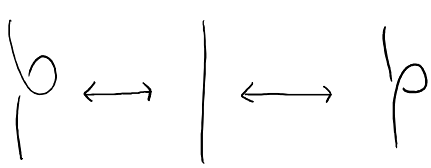
(RII) =

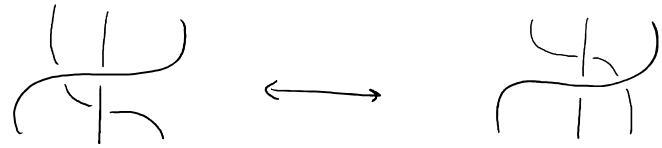


With orientation,

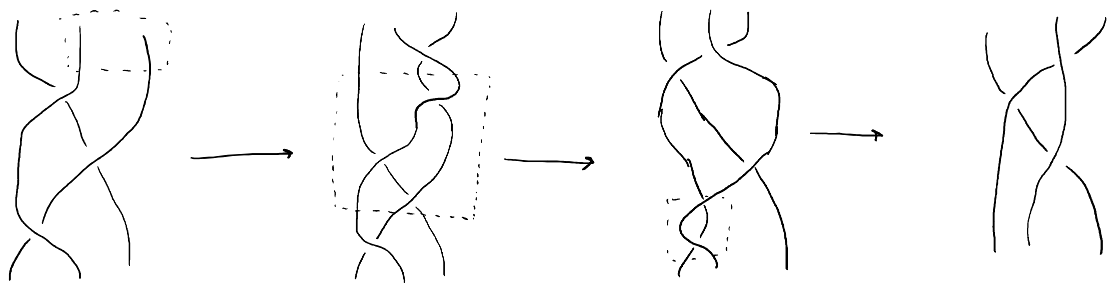
X_+ =  , X_- =  , Y_+ =  , Y_- = 

(RI) =



(R III) 

Some other variants of R III are already implied:



Theorem (Reidemeister 1927, Alexander Briggs 1926)

Two links L, L' w/ general position diagrams $\hat{L}, \hat{L}' \in \mathbb{R}^2$ are equivalent iff there is a sequence of link diagrams

$$\hat{L} = \hat{L}_0 \longrightarrow \hat{L}_1 \longrightarrow \cdots \longrightarrow \hat{L}_N = \hat{L}'$$

s.t. $\hat{L}_j$ is obtained from $\hat{L}_{j-1}$ by application of an RI, RII or RIII move or planar isotopy.

So:

$$\{\text{Tame links in } S^3\} / \text{ambient isotopy}$$

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$$\Big\updownarrow \text{ bijection}$$

$$\{\text{Gen. Pos Link diagrams}\}/\ R0, RI, RII, RIII.$$

Variation for Oriented Links exists

Framed Links

$\hat{L}$ represents unique
framed link class
via blackboard framing.

$$L_\nu \underset{\text{component}}{\subset} L \longrightarrow w_\nu = \text{writhe}$$

$$\longrightarrow \mu_\nu = \text{winding } \#$$

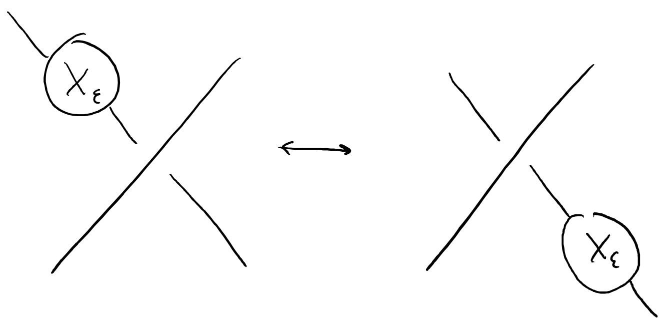
$\vec{\gamma}$ = path for $\hat{L}_\nu$ in $\mathbb{R}^2$ $\qquad K_\nu : t \longmapsto \frac{1}{\|\frac{d\vec{\gamma}}{dt}\|} \frac{d\vec{\gamma}}{dt} \in S^1$

$K_\nu : S^1 \longrightarrow S^1$ gives winding #

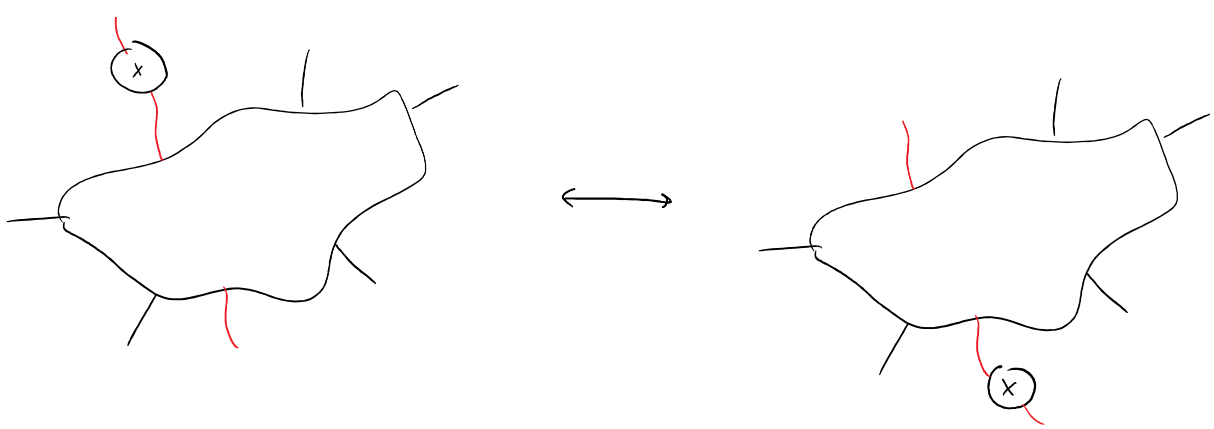
Z	X_+	X_-	Y_+	Y_-
$w_\nu(L \#_\nu Z) - w_\nu(L)$	+1	−1	+1	−1
$\mu_\nu(L \#_\nu Z) - \mu_\nu(L)$	−1	−1	+1	+1

* w_ν, μ_ν do not change under RII or RIII.

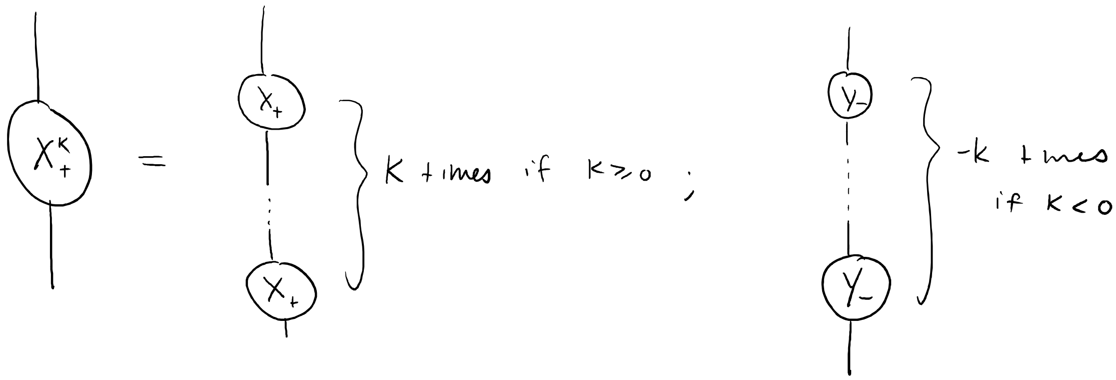
* RII & RIII-moves imply



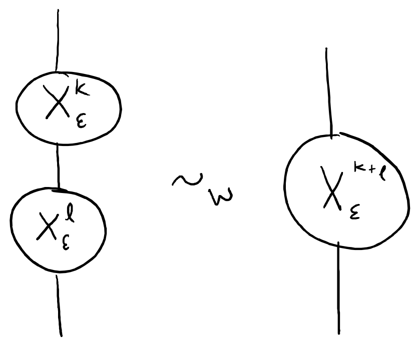
more generally:

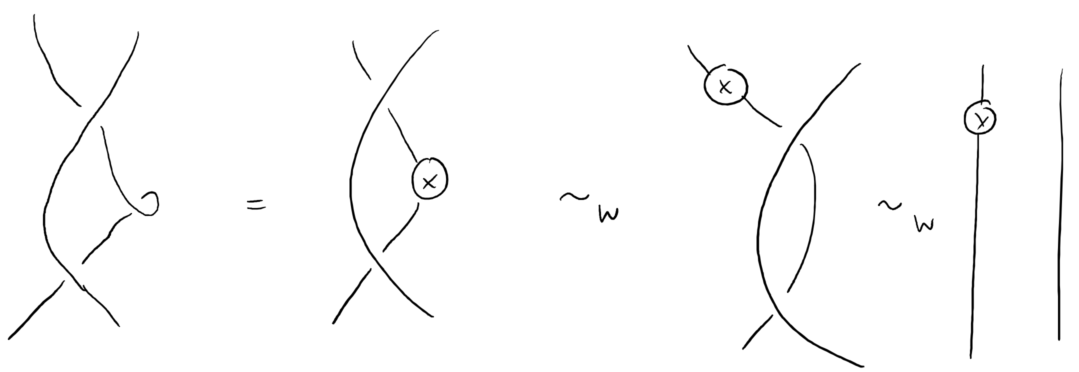


$\sim_w$ = equivalence via only RII and RIII-moves



so



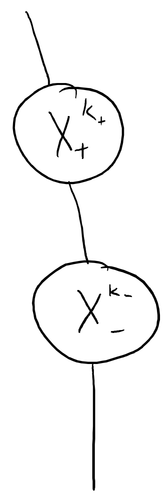


L, L' equiv framed links

$$\hat{L} = \hat{L}_0 \to \cdots \to \hat{L}_N = \hat{L}'$$

post pone all RI moves

$$\hat{L} = \hat{\tilde{L}}_0 \xrightarrow[\sim_w]{} \hat{\tilde{L}}_1 \to \cdots \to \hat{\tilde{L}}_M \xrightarrow{RI} \hat{L}'$$

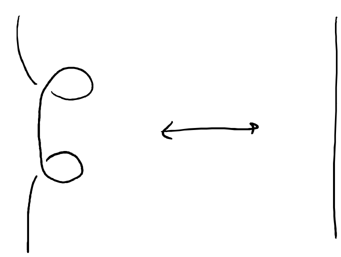


$$\Delta w = k_+ - k_-$$

$$\Delta \mu = -(k_+ - k_-)$$

On each component, $k_+ = k_-$

So (RI^{fr}) =



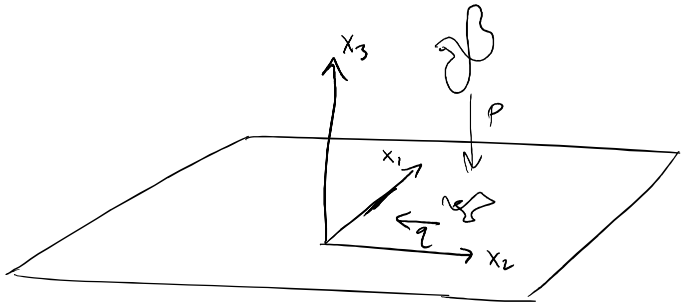
Thm L, L' equivalent as framed links, $\hat{L}, \hat{L}'$ gen. pos projections. $\rightsquigarrow$ giving L, L' by blackboard framing.
Then $\exists$ sequence of $R0$, RI^{fr}, RII, $RIII$ moves changing $\hat{L}$ to $\hat{L}'$.

Smooth World

$$\varphi : S^1 \hookrightarrow \mathbb{R}^3 \xrightarrow{p} \mathbb{R}^2$$

$$\phi : S \times I \hookrightarrow \mathbb{R}^3$$

"functions that are transverse is an open & dense set".

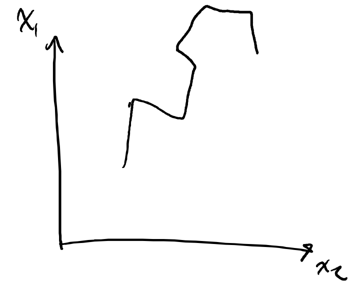


$\hat{L}$: PL- link diagram is in gen. pos wrt

$$q : (x_2, x_1) \longmapsto x_1$$

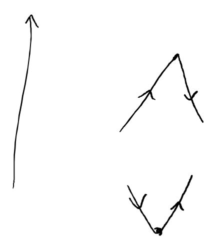
if $q\big|_{\bar{s}} : \bar{s} \longrightarrow \mathbb{R}$ is injective

segment of knot.



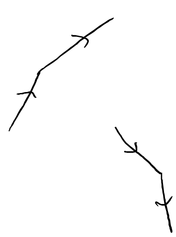
Given orientation on $\hat{L}$,

each segment is either increasing or decreasing



local max or

local min



$\longrightarrow$ straighten