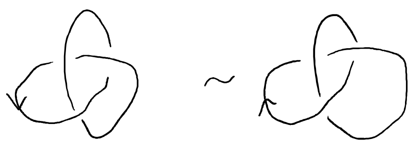
8/28

Wednesday, August 28, 2019 15:01

$i : S^1 \hookrightarrow S^3$

$r : S^1 \longrightarrow S^1$

$z \longrightarrow \bar{z}$ (go the opposite direction)



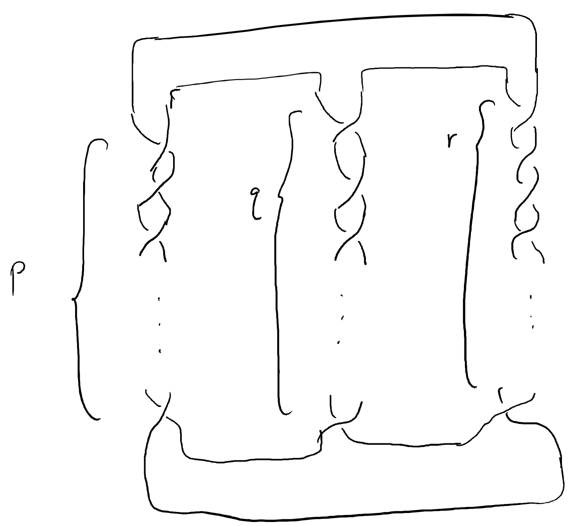
, but 8_{17} doesn't do this.

"invertible"

not invertible

Knot Symmetries

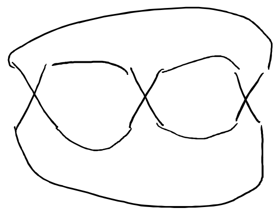
Pretzel-Knots



$K(p,q,r)$

$K(0,0,0)$ is not a knot.

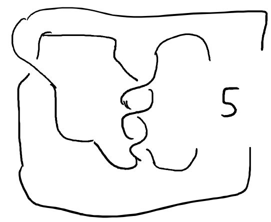
if p, q, r are odd then



it's a knot.

(depends only on parity of p, q, r).

$K(-1, 3, 5) =$



<u>Thm</u> [Trotler 1960]:

If p, q, r are odd & distinct & > 1, then $K(p, q, r)$ is non-invertible.

$$\pi_K = \pi_1 (S^3 \setminus K)$$

Crossing #	% non-invertible
3 - 7	0 %
8	5 %
9	4 %
10	20 %
11	34 %
12	52 %
13	62 %
14	81 %
15	89 %
16	94 %

$$K \rightsquigarrow K^{-1}$$

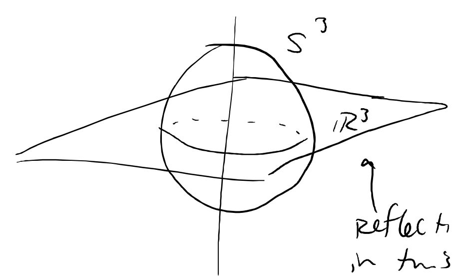
invertible $K \approx K^{-1}$

$[K] = [K^{-1}] \leftarrow$ ambient isotopy class.

Fix an orientation reversing homeomorphism

$$r : S^3 \longrightarrow S^3$$

eg



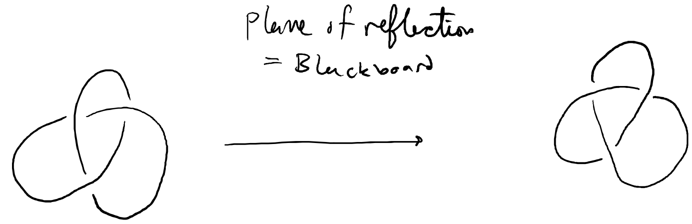
reflection in this plane.

mirror of K:

$$\overline{K} = r(K).$$

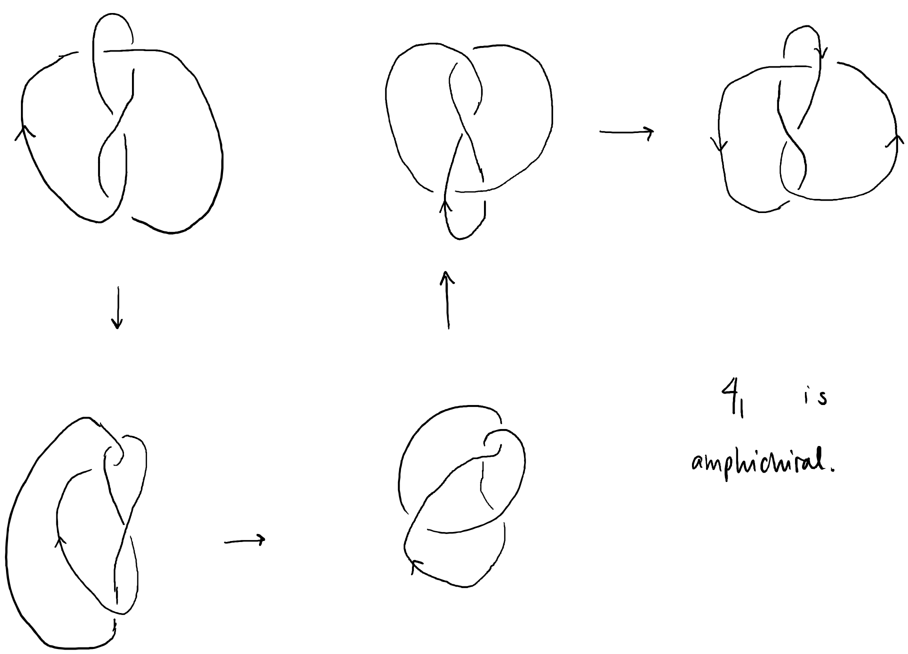
$i : S^1 \hookrightarrow S^3 \ , \quad \bar{i} = \mathcal{H} \circ i \ .$

3_1



$\overline{3_1}$

Def K is <u>amphichiral</u> if $K \simeq \overline{K}$ $\quad ([K] = [\overline{K}])$.



4_1 is amphichiral.

If $K \not\simeq \overline{K}$ then K is chiral.

Crossing #	% knots that are amphichiral
3	0 %
4	100 %
5	0 %
6	67 %
7	0 %
8	24 %
9	0 %
10	7.9 %
11	0 %
12	2.7 %
13	0 %
14	0.58 %
15	0.000395 % ← disproof of conjecture!

Symmetry Types

$\mathbb{Z}/_2 \times \mathbb{Z}/_2$ acts on $\{[K]\}$

gen: i ↑ inversion, m ↑ mirror

$$i[K] = [K^{-1}]$$

$$m[K] = [\bar{K}]$$

$\mathrm{Stab}([K])$

—

* Fully amphichiral : $\mathrm{Stab}([K]) = \mathbb{Z}/2 \times \mathbb{Z}/2$

  $(K \approx \bar{K} \approx K^{-1} \approx \bar{K}^{-1})$

  eg: 4_1

* Negative (−) Amphichiral : $\mathrm{Stab}([K]) = \langle m \cdot i \rangle$
  Non-invertible

  $(K \approx \bar{K}^{-1} \not\approx \bar{K} \approx K^{-1})$

  eg: 8_{17}

* Positive (+) Amphichiral : $\mathrm{Stab}([K]) = \langle m \rangle$
  Non-invertible

  $(K \approx \bar{K} \not\approx K^{-1} \approx \bar{K}^{-1})$

  eg: some 12+ crossing knot: 12a427

* Chiral, invertible : $\mathrm{Stab}([K]) = \langle i \rangle$

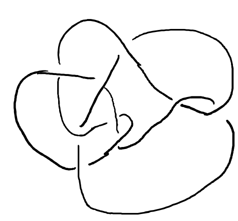
  $(K \approx K^{-1} \not\approx \bar{K} \approx \bar{K}^{-1})$

  eg: 3_1

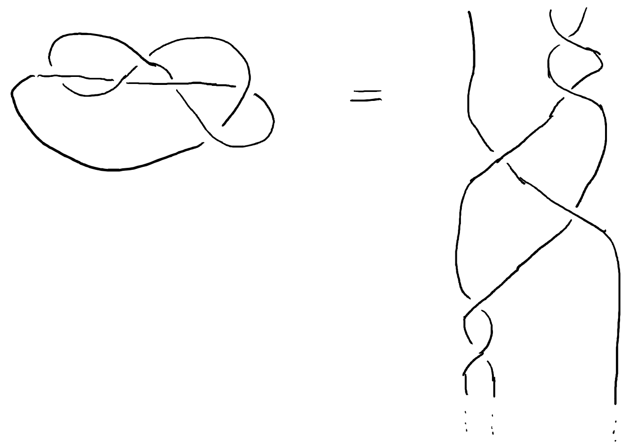
* Fully chiral : $\mathrm{Stab}([K]) = 1$
  (Chiral, non-invertible)

  all $K, \bar{K}, K^{-1}, \bar{K}^{-1}$ distinct

  eg: 9_{32} :



Exercise: Show 6_3 is fully amphichiral



$$\left\{ \begin{array}{l} h : S^3 \circlearrowleft \\ h' : S^3 \circlearrowleft \quad \text{gives diff notion of chirality?} \end{array} \right.$$

orient. reversing homeomorphisms.

$$\underbrace{h' \circ h^{-1}}_{= f} \quad \text{is} \quad \text{orientation-preserving}.$$

$$h' = f \circ h$$

<u>Theorem</u> [Fischer 1960]

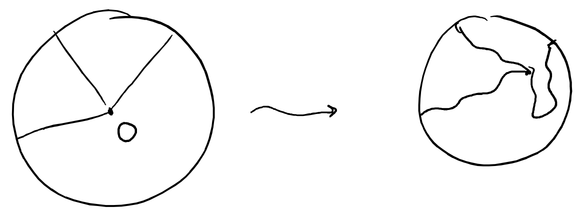
Suppose $f : S^3 \to S^3$ is an orientation-preserving (PL) homeomorphism. Then f is (PL) isotopic to id_{S^3}.

That is, $\exists\, F : S^3 \times I \longrightarrow S^3$, F_t = homeom, $F_0 = f$, $F_1 = id_{S^3}$.

Thm Let $f : \underset{\text{ball}}{B^n} \longrightarrow B^n$ homeom s.t. $f|\partial B_n = \text{identity}$.

Then f is isotopic to id_{B^n}.

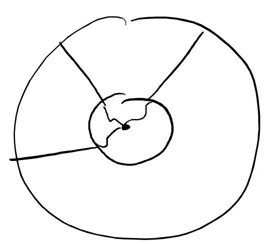
$0 \in B^n \subset \mathbb{R}^n$



~~pf~~ alexander trick:

$F_t : B^n \longrightarrow B^n$

$$F_t(x) = \begin{cases} t\, f(\tfrac{1}{t}x) & \text{if } x \in tB^n \\ x & \text{otherwise} \end{cases}$$



Thm Two knots K_0 and K_1 are equivalent if and only if and only if there is an orientation-preserving $f: S^3 \longrightarrow S^3$ s.t. $f(K_0) = K_1$, preserving knot orientation.