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Monday, August 26, 2019 14:59

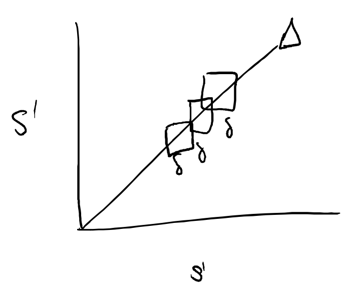
* Knot is <u>tame</u> if it is ambient isotopic to a piecewise-linear knot.

* Every PL-knot is ambient isotopic to a smooth knot:

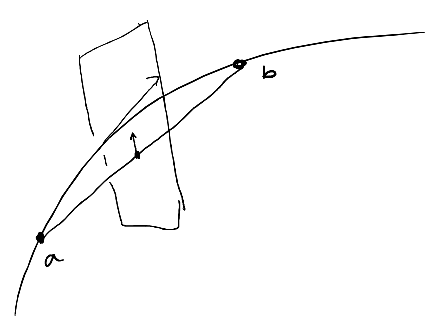
* Every $\underbrace{\text{Smooth knot}}_{S^1 \xrightarrow{\gamma} \mathbb{R}^3}$ is ambient isotopic to a PL-knot.

$$S^1 \times S^1 \longrightarrow S^2$$

$$(a, b) \longmapsto \begin{cases} \dfrac{\gamma(a) - \gamma(b)}{\|\gamma(a) - \gamma(b)\|} & a \neq b \\[2ex] \dfrac{\gamma'(a)}{\|\gamma'(a)\|} =: \hat{\gamma}(a) & a = b \end{cases}$$



$$\Delta = \{(a,a) : a \in S^1\}$$



<u>Lemma</u>: A knot is tame iff it is ambient isotopic to a smooth knot.

Isotopy Extension Theorem:

Suppose M and N are smooth manifolds and N is compact.

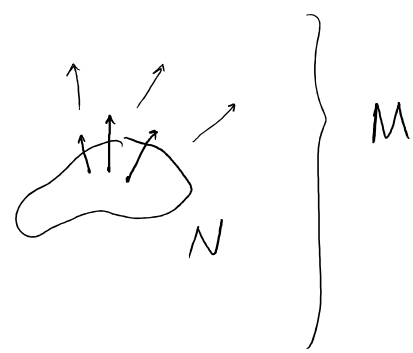
Suppose $h : N \times I \longrightarrow M$ is a smooth isotopy.
$(x, t) \longmapsto h_t(x)$

Then h extends to an ambient isotopy $\Phi : M \times I \longrightarrow M$

$$\left(\Phi_0 = id, \quad h_t = \Phi_t \circ h_0 \right)$$

$$\frac{\partial}{\partial t} h_t(x)$$

(Hirsch, Chapter 9)



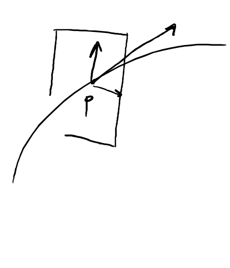
Key here is "smooth."

In C^∞-world, isotopy $\Longleftrightarrow$ ambient isotopy.

Tubular neighborhoods of Knots

Normal bundle

$\mathbb{R}^3$



E_p = directions $\perp$ to tangent at p.

$$\bigcup_{p \in K} E_p = \nu_M(K)$$

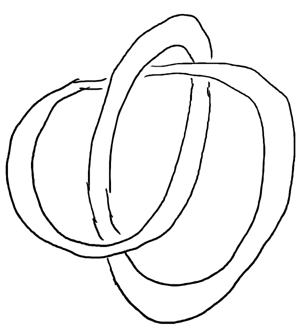
<u>Tubular nhd Thm</u>: $N \hookrightarrow M$

There is a diffeom. $\nu_M(N) \longrightarrow \underbrace{N(N)}_{\text{open nhd of } N} \subset M$

$$\nu_M(K) \longrightarrow N(K) \subset \underset{\substack{\shortparallel \\ \text{oriented}}}{S^3}$$

In oriented case,

$$\nu_M(S^1) \cong \mathbb{R}^2 \times S^1 .$$



$$\longleftarrow \underset{\text{open disk}}{D^2} \times S^1$$

where $0 \times S^1 \longrightarrow$ the knot.

$$\begin{array}{c} f_0, f_1 : \mathbb{R}^2 \times S^1 \longrightarrow \nu_M(S^1) \\ \searrow \quad \swarrow \\ S^1 \end{array}$$

$$
\begin{tikzcd}
\mathbb{R}^2 \times S^1 \arrow[rr, "f_1^{-1} f_0"] \arrow[dr] & & \mathbb{R}^2 \times S^1 \arrow[dl] \\
& S^1 &
\end{tikzcd}
$$

$$(v, \theta) \longmapsto (\mu(\theta) v, \theta)$$

Where $\mu : S^1 \longrightarrow GL^+(\mathbb{R}^2)$.

$$GL^+(\mathbb{R}^2) \cong S^1 \times \mathbb{R}^3$$

$\rightsquigarrow$ winding # for μ.

$$
\begin{array}{ccc}
\mu^* : H_1(S^1) & \longrightarrow & H_1(GL^+(\mathbb{R}^3)) \\
\| & & \| \\
\mathbb{Z} & \longrightarrow & \mathbb{Z} \\
n & \longmapsto & \text{winding\#} \cdot n .
\end{array}
$$

$\mathbb{R}^2 \times S^1 \xrightarrow{J \circ f} S^3$

$\bullet \times S^1 \xrightarrow[\text{knot}]{} S^3$

$\odot \times S^1 \longrightarrow S^3$

$\odot \times S^1 \xrightarrow[\text{tubular nhd}]{} S^3$

 $\times S^1 \longrightarrow S^3$

$\uparrow$

something that twists around the knot. it winds around the core (corresponding to ______)

 $\times S^1 \xrightarrow[\text{Ribbon}]{} S^3$

Def a framing of a knot is an isotopy class of trivializations of its normal bundle.

Lemma: if the relative winding # of two trivializations is 0, then the framings are the same.

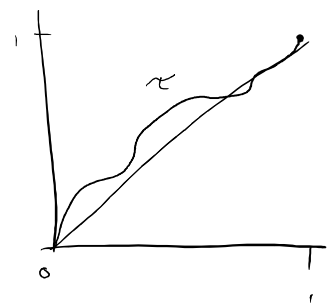
Cor. Every tame knot admits a topological tubular neighborhood.

Lemma Two knots that only differ by parameterization w/ same orientation are equivalent.

$K_1, K_2 : S^1 \hookrightarrow S^3$

where $K_1 = K_0 \circ \tau$ where $\tau : S^1 \to S^1$ orientation preserving homeomorphism.

eg



$sx + (1-s)\tau(x)$

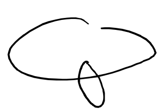
homotopes τ to id.

τ is isotopic to id on S^1

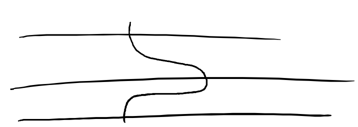
τ_s.

$K_s = K_0 \circ \tau_s$ is an isotopy.

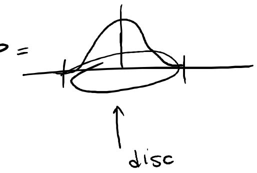
J_0 is tub. nhd of K_0

 $\longrightarrow S^3$

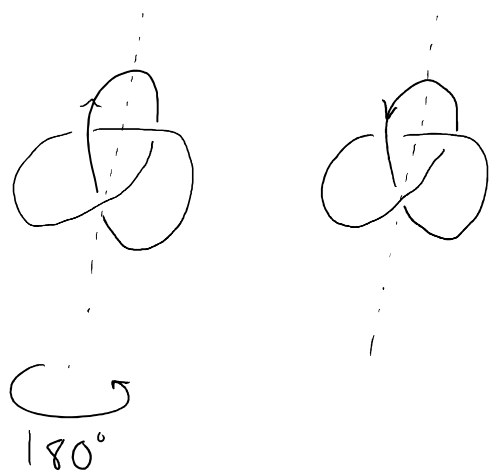
Shift along tube to extend to ambient isotopy.



$$G((\theta, v), s) = (\tau(\theta, s\rho(v)), v)$$

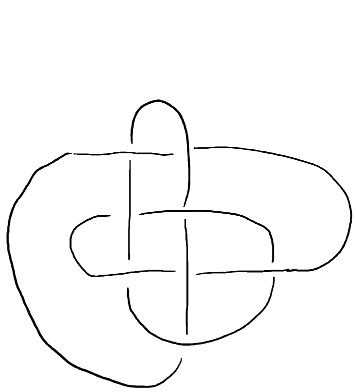
where $\rho =$ 

if τ isn't orientation preserving, the above argument doesn't work



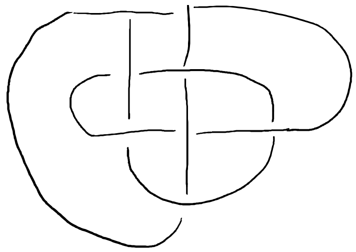
this knot is reversible: it's equivalent to the one w/ reversed orientation.

aka invertible



8_{17} is <u>not</u> reversible. (invertible)

this is the first one (of many)



˘17

(invertible)

this is the first one

(of many)