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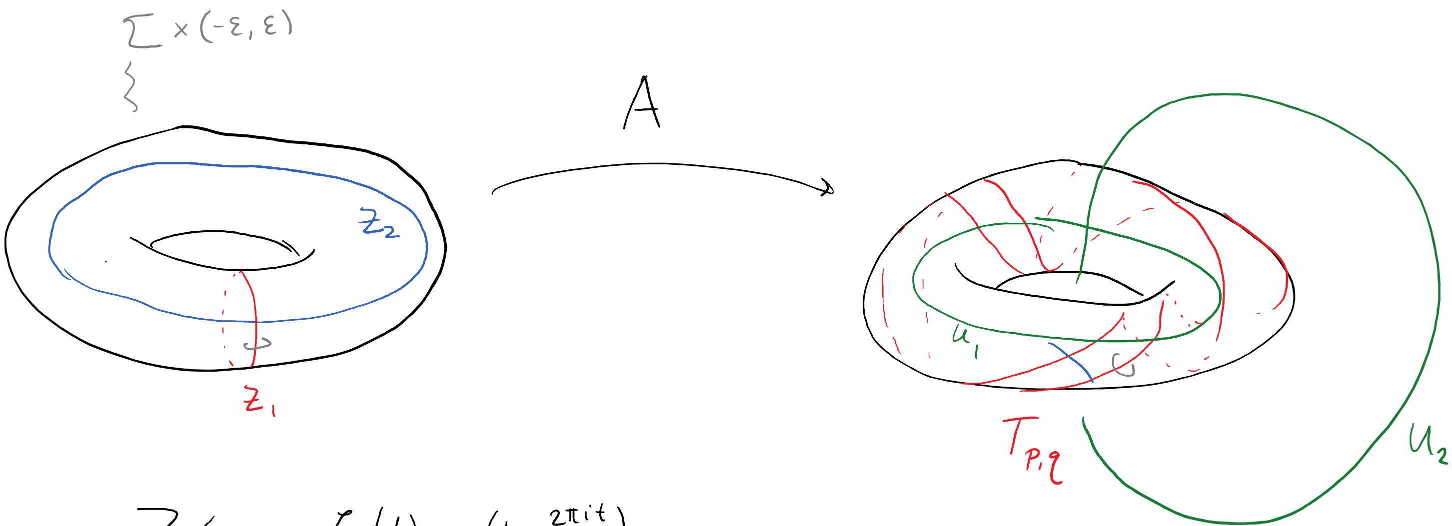
$$A = \begin{bmatrix} p & s \\ q & r \end{bmatrix}, \quad pr - qs = 1$$



$$Z_2 \hookleftarrow \zeta_2(t) = (1, e^{2\pi i t}) \qquad A \circ \zeta_2 = (e^{2\pi i s t}, e^{2\pi i r t})$$

$$Z_1 \hookleftarrow \zeta_1(t) = (e^{2\pi i t}, 1) \qquad \underbrace{A \circ \zeta_1 = (e^{2\pi i p t}, e^{2\pi i q t})}_{T_{p,q}}$$

$$\pi_{T_{p,q}} = \langle u_1, u_2 \mid u_1^q = u_2^p \rangle$$

In left curve, meridian curve is

$$\tau \simeq (Z_2^-)^{-1} * Z_2^+$$

(Z_2^-: pushed in; Z_2^+: pushed out)

A maps Z_2 to $\tilde{\mu}$

Parameterized by $A \circ \int_2$

A maps $Z_2^{\pm}$ to $\tilde{\mu}^{\pm}$

In right picture, meridian curve $\simeq (\tilde{\mu}^-)^{-1} * \tilde{\mu}^+$

Push $\tilde{\mu}^+$ into H_2, param by $(e^{2\pi i s t}, e^{2\pi i r t})$

$\rightsquigarrow (e^{2\pi i s t}, 0)$

Class given by u_2^s.

In H_1, for $\tilde{\mu}^-$ rep. by u_1^r

So <u>Lemma</u> $\tau^{-1} = u_1^{-r} u_2^s$ is meridian generator of $\pi_{T_{p,q}}$.

$$\pi_{T_{p,q}} \longrightarrow H_1(S^3 \setminus T_{p,q}) = \mathbb{Z}$$

$$u_1 \longmapsto p$$

$$u_2 \longmapsto q$$

$$z := u_1^q = u_2^p .$$

Then z is central in $\pi_{T_{p,q}}$.

$$\pi_{T_{p,q}} / \langle z \rangle = \langle u_1, u_2 \mid u_1^q = u_2^p = 1 \rangle$$

$$= \langle u_1 \mid u_1^q \rangle * \langle u_2 \mid u_2^p \rangle$$

$$= (\mathbb{Z}/q) * (\mathbb{Z}/p)$$

<u>Corollary</u> $\langle z \rangle$ is the whole center of $\pi_{T_{p,q}}$.

$G = \pi_{T_{p,q}}$,

$p,q > 0, \ p',q' > 0$

$$G/Z(G) = \mathbb{Z}/p * \mathbb{Z}/q \cong \mathbb{Z}/p' * \mathbb{Z}/q'$$

$$\Rightarrow \quad \{p,q\} = \{p',q'\}.$$

Thus $T_{p,q} \not\approx T_{p',q'}$ if $\{|p|,|q|\} \neq \{|p'|,|q'|\}$.

$$S^3 \longrightarrow S^3$$

$(z,w) \longmapsto (w,z)$ $\longleftarrow$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \in SO(4)$

$(z,w) \longmapsto (\bar{z},\bar{w})$ $\longleftarrow$ $\begin{bmatrix} \begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix} & 0 \\ 0 & \begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix} \end{bmatrix} \in SO(4)$

So:

$$T_{p,q} \approx T_{q,p}$$

$$T_{p,q} \approx T_{-p,-q}$$

Remaining: $T_{p,q} \overset{?}{\approx} T_{p,-q}$

$$(z,w) \longmapsto (z,\bar{w}).$$

No, $T_{p,q}$ is the mirror of $T_{p,-q}$

(it turns out that $T_{p,q}$ are all chiral).

Classification of Torus Knots: $(|p|, |q| > 1$, coprime$)$

$$T_{p,q} \approx T_{p',q'} \quad \text{iff}$$

$$(p', q') \in \left\{ \begin{matrix} (p,q), & (-p,-q), \\ (q,p), & (-q,-p) \end{matrix} \right\}.$$

$T_{p,q}$: * invertible

* chiral (Jones Polynomial)

Recall if K is composite, then

$$Z(\pi_K) \subset \langle \tau \rangle$$

↑ meridian generator

For $T_{p,q}$, $u_1^q \overset{\text{assume}}{=} \tau^m = (u_1^{-r} u_2^s)^m$

Eg $\pi_{T_{p,q}} \longrightarrow \mathbb{Z}/_p * \mathbb{Z}/_q$

($\bar{u}_2 \mapsto \mathbb{Z}/_p$, $\bar{u}_1 \mapsto \mathbb{Z}/_q$)

$\rightsquigarrow (\underbrace{\bar{u}_1^{-r} \bar{u}_2^s}_{\text{nontrivial}})^m = 1$ in $\mathbb{Z}/_p * \mathbb{Z}/_q$

$\rightsquigarrow \quad m = 0$

So $u_i^i = 1$, contradiction.

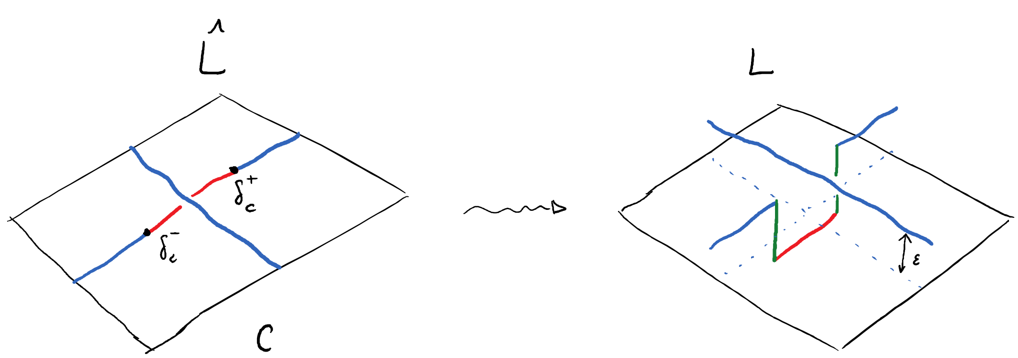
Cor: $T_{p,q}$ is prime.

Cor: There are ∞-many inequivalent prime knots.

Wirtinger Presentation

$\tilde{L} \hookrightarrow \mathbb{R}^3 \quad \text{proj} \quad \hat{L} \hookrightarrow \mathbb{R}^2 = \mathbb{R}^2 \times \{0\} \hookrightarrow \mathbb{R}^3.$

Reconstruct $L \approx \tilde{L}$ from $\hat{L}$.



$H_+ = \mathbb{R}^2 \times \mathbb{R}_{>0}$

$$H_- = \mathbb{R}^2 \times \mathbb{R}_{<0}$$

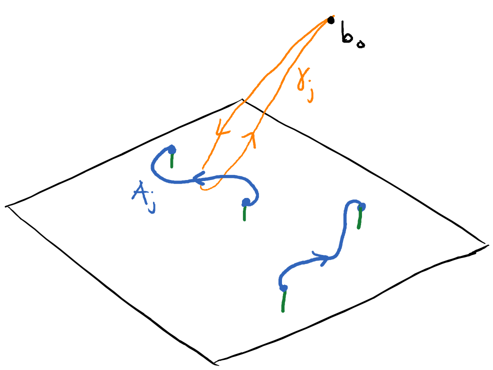
$$\hat{H}_\pm = \overline{H}_\pm \setminus L.$$

So $\mathbb{R}^3 \setminus L = \hat{H}_+ \cup \hat{H}_-$

↑

over $\mathbb{R}^2 \setminus \{ \delta_c^\pm \mid c \text{ crossing in } \hat{L} \}$

$\overline{H}_+ \cap L$ = blue + part of green



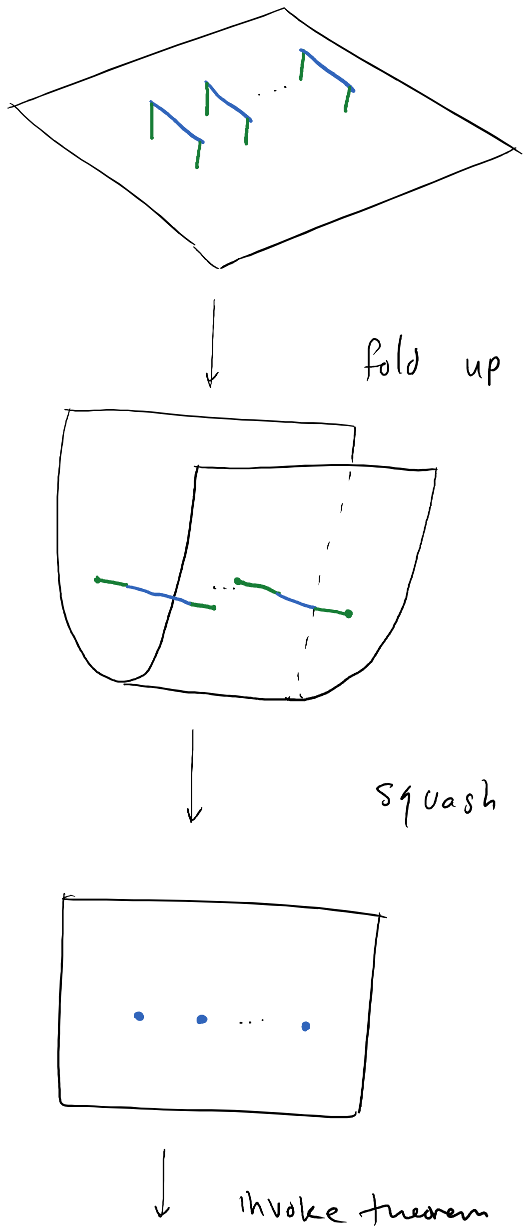
Let $\{A_j\}$ = set of maximal blue arcs.

set $x_j = [\gamma_j] \in \pi_1(\hat{H}_+, b_0)$

<u>Claim:</u> $\pi_1(\hat{H}_+, b_0)$ is freely gen by $\{x_j\}$

$$\cong F_N = \langle x_1, \dots, x_N \rangle$$

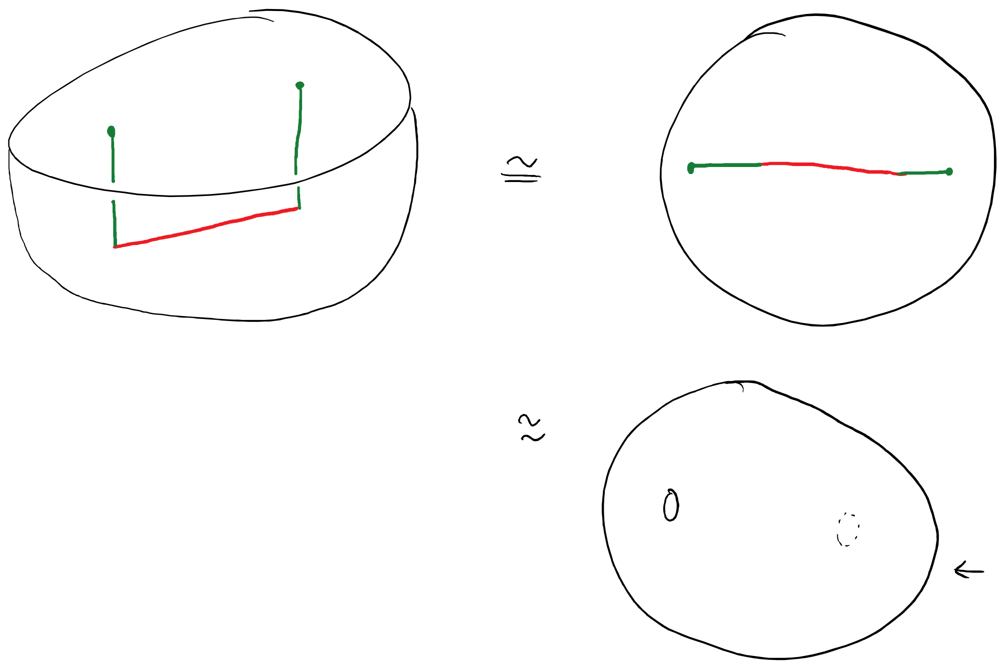
Pf: isotop arcs to parallel lines



$$\pi_1 = F_N .$$

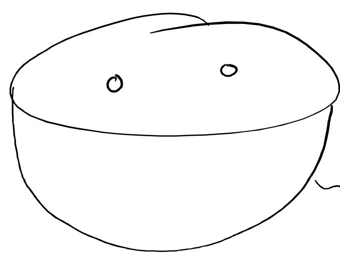
□

Now attaching crossings (a n.h. of green & red is removed)



← sphere w/ two holes

So instead just glue in



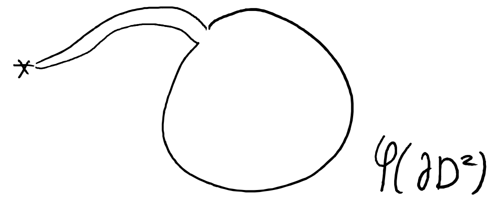
only the bottom must be attached, the top is already there

So just glue in a disc around each crossing.

<u>Recall</u> X, $\varphi : \partial D^2 \longrightarrow X$

$$\pi_1 \left(X \underset{\varphi}{\cup} D^2 \right) = \pi_1(X) / [\varphi]$$

When ζ = path



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