Markov Thm $\hat{cl} : \bigcup_n B_n \Big/ {\sim_M} \longrightarrow \{\text{classes of links}\}$ is bijection.

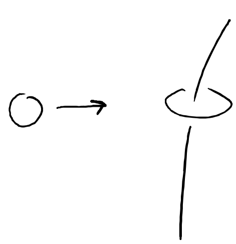
$\sim_M$: Markov Move & Conjugation

Knot Groups & Torus Knots

$L \subset S^3$; $\quad \pi_L := \pi_1(S^3 \setminus N(L), x_0)$

$L = K_1 \sqcup \cdots \sqcup K_m$ (m-comp)

$$1 \longrightarrow [\pi_L, \pi_L] \longrightarrow \pi_L \longrightarrow \mathbb{Z}_{[\mu_1]} \oplus \cdots \oplus \mathbb{Z}_{[\mu_m]} \longrightarrow 1$$



$[\mu_j] \in [S^1, S^3 \setminus N(L)]$

$\left([S^1, X] = \text{set of conjugacy classes of } \pi_1(X, x_0). \right)$

$M_j =$ conjugacy class of $[\mu_j]$

for knots: $\quad S_K = S^3 \setminus N(K)$

$$M_K \subset \pi_K = \pi_1(S_K), \quad \pi_K' = [\pi_K, \pi_K]$$

$$\begin{tikzcd} 1 \arrow[r] & \pi_K' \arrow[r] & \pi_K \arrow[r] & \mathbb{Z} \arrow[r] \arrow[l, bend right] & 1 \\ & & x \arrow[r, mapsto] & 1 & \end{tikzcd}$$

Have form

$$\pi_K = \mathbb{Z}_\tau \ltimes \pi_K' \quad , \quad \tau \in M_K .$$

$$\pi_{\text{unknot}} = \mathbb{Z} \iff \pi_K \text{ abelian.}$$

<u>Thm</u> $\quad \pi_K = \mathbb{Z} \iff K = \text{unknot}.$

<u>Loop Theorem</u>

Let $M = 3$-manifold w/ boundary (not necessarily compact)

Loop Theorem

Let $M = 3$-manifold w/ boundary (not necessarily compact or orientable).

$$f : (D^2, \partial D^2) \longrightarrow (M, \partial M)$$

for which $f|_{\partial D^2}$ is <u>not</u> null-homotopic in ∂M.

Then there is embedding w/ same properties.

(Papakyriakopoulos '86)

<u>Cor</u> Dehn's Lemma

If $C \hookrightarrow \partial M$, with C null-homotopic in M.
C = circle

Then C bounds an embedded disc in M

<u>pf</u> peel the apple except near C.

<u>Cor</u> : $\pi_K = \mathbb{Z} \Rightarrow K$ bounds a disc

<u>pf</u> λ = 0-framed longitude

$\Rightarrow \lambda$ = null homotopic

λ bounds a disc in $S^3 \setminus N(K)$

$\rightsquigarrow$ attach it to K.