10/16

Wednesday, October 16, 2019 15:00

Alexander Modules

K knot

$$\pi_K \triangleright \pi_K^{(1)} \triangleright \pi_K^{(2)} \triangleright \cdots$$

$$\pi_K^{(i+1)} = [\pi_K^{(i)}, \pi_K^{(i)}]$$

$$\pi_K / \pi_K^{(1)} \circlearrowright \pi_K^{(1)} / \pi_K^{(2)}$$

To $\pi_K^{(1)} \triangleleft \pi_K$ Find regular covering

$$\begin{array}{ll} \tilde{p} & \tilde{X} \\ & \big\downarrow p \\ p & X \end{array} \quad \text{connected}$$

$$1 \longrightarrow \pi_1(\tilde{X}, \tilde{p}) \xrightarrow{p_*} \pi_1(X, p) \xrightarrow{\lambda} \mathcal{D} \longrightarrow 1$$

$\mathcal{D}$ = gp of Deck transformations

$$= \{ \psi : \tilde{X} \circlearrowleft \, : p \circ \psi = p \}$$

$$\tilde{p} \in \mathcal{F} = p^{-1}(p)$$

$$\mathcal{D} \longrightarrow \mathcal{F}$$

$$g \longmapsto g(\tilde{p})$$

is a bijection.

$g : \mathcal{F} = p^{-1}(p)$
$\circlearrowleft$

for $X = S_K = S^3 - K$,

$\tilde{X}$ is the ∞-cyclic cover

that is $\mathcal{D} = \mathbb{Z} \cong H_1(S_K) = \pi_K / \pi_K'$

$$\pi_1(\tilde{X}, \tilde{p}) = \pi_1(\tilde{S}_K) = [\pi_K, \pi_K] = \pi_K'$$

$$\pi_K^{(1)} / \pi_K^{(2)} = H_1(\tilde{X})$$

$\mathcal{D}$ acts on $\tilde{X}$ with $\mathcal{D}$ generated by

$$\tau : \tilde{X} \longrightarrow \tilde{X}$$

$$t = \tau_* : H_1(\tilde{X}) \longrightarrow H_1(\tilde{X})$$

$$\underbrace{3t^9 + 7t^5 - 4t^{-3}}_{\in \text{ Group Algebra}} : H_1(\tilde{X}) \longrightarrow H_1(\tilde{X}).$$

$\mathbb{Z}[t, t^{-1}]$ laurent polynomials

<u>Def</u> the Alexander module for a knot K is

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$H(\tilde{S}_K)$ where $\tilde{S}_K$ is ∞-cyclic cover of S_K.

as a $\mathbb{Z}[t, t^{-1}]$ module w/ action of t given by Deck transf.

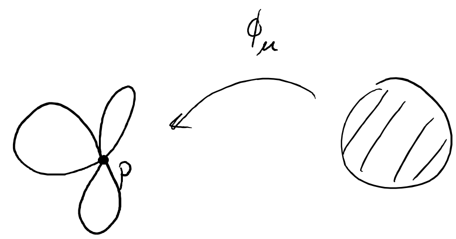
* for links: $H_1(\tilde{S}_L)$ as $\mathbb{Z}[t_1, t_1^{-1}, \dots, t_k, t_k^{-1}]$

Describe $H_1(\tilde{X})$ as $\mathbb{Z}[\mathcal{D}]$-module.

Given $\pi_1(X, p) = \langle s_1, \dots, s_n \mid r_1, \dots, r_m \rangle$

* consider X as CW-complex

$X^0 = \{p\}$



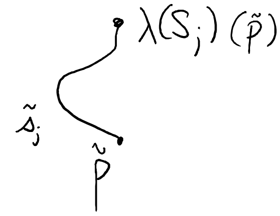
$X^1 = \{e_j^1\}_{j=1}^n$

$X^2 = \{e_\mu^2\}_{\mu=1}^m$ $\qquad \phi_\mu\big|_{\partial D} = \varphi_\mu^* : (S^1, 1) \longrightarrow (X, p)$

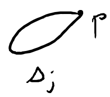
Cell structure $\tilde{X}$ given by lifting cells in X.

$\tilde{X}^0 = \mathcal{F} = p^{-1}(p)$

Denote lift of s_j based at $\tilde{p}$ by $\tilde{s}_j$



$[s_j] = S_j$
$\lambda(S_j) \in \mathcal{D}$



lift of s_j at $g(\tilde{p})$ given by $g \circ \tilde{s}_j$. $\quad (g \in \mathcal{D})$.

parametrize all 1-cells $e'_{gj} = g e'_j$.

2-cells $e^2_{1,\mu}$

Describe $H_1(\tilde{X}, \mathbb{Z})$ in cellular homology.