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<u>Theorem</u> $U_\hbar(sl_2)$ is a quasitriangular Hopf algebra.

$\rightarrow (U_\hbar(sl_2), \Delta, \varepsilon, S)$ Hopf alg.

$\rightarrow$ Drinfeld pairing $(\cdot,\cdot)$ $\quad \underset{\{H,F\}}{\underset{\uparrow}{U^{\leq 0}}}, \underset{\{H,E\}}{\underset{\uparrow}{U^{\geq 0}}} \subseteq U$ Hopf algs

$R \in U^{\leq 0} \otimes U^{\geq 0} \subset U \otimes U$ canonical elt of pairing

satisfies Cabling ids

$$\longrightarrow \quad R = q^{\frac{H\otimes H}{2}} \exp_q\left((q-q^{-1})\, F \otimes E\right)$$

where $\exp_q(x) = \sum_{n\geq 0} q^{n(n-1)/2} \dfrac{x^n}{[n]!}$

$$\exp_q(qx) - \exp_q(q^{-1}x) = (q-q^{-1})\, x \exp_q(qx)$$

Prop : $\forall\, x \in U_\hbar(sl_2)$, $\quad R\,\Delta(x)\,R^{-1} = \Delta^{op}(x)$.

use $\quad [E, F^n] = \frac{[n]}{q-q^{-1}}\left(q^{n-1}K - q^{-n+1}K^{-1}\right)F^{n-1}$.