Lec 11/6

Monday, November 6, 2017 14:26

Handouts Anything Classical, Not too long can be there.

Chapter 6

Log 2 expansion 311

formula 6.1 $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$

Walliz formula.

Pg 344 $\sum \frac{1}{n^{2k}} = q\,\pi^{2k}$ $\sum \frac{1}{n^3}$ is irrational. (Apéry)

$\int_0^1\int_0^1 \frac{dx\,dy}{1-xy}$ exercise: use this integral to show $\sum \frac{1}{n^2} = \frac{\pi^2}{6}$

$\frac{\pi^2}{6}$ product on 338

Theorem 6.17.

6.8 \ 6.8.3

Chapter 7

Thms 7.1, 7.2 (transformation rules)

↳ 7.3

Know formulation of 7.15

(Prove 3 ways that e is irrational)

(§ 7.16, Taylor expansion, Continued fraction $\frac{e-1}{e+1}$)
etc
or Legendre H&W thing.

7.47, 7.50 for $e \in \mathbb{Q}$.

Liouville Thm / numbers $\sum \frac{(-1)^n}{10^{n!}}$

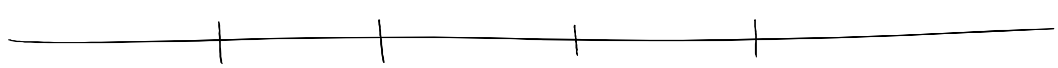
Theorem 7.34 (know proof)

Number-Theoretic Functions and the Distribution of Primes

Maybe not

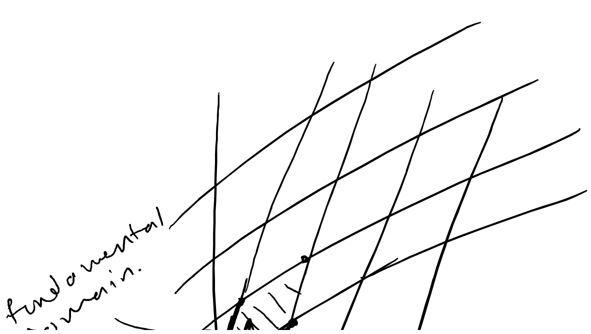
exercise: Let $L \subset \mathbb{R}^n$ be a lattice. Let Δ be the volume of its fundamental domain. If $S \subset \mathbb{R}^n$ and $\mathrm{vol}(S)$ $(\mu(S))$ satisfies $\mathrm{Vol}(S) > \Delta$ then $S - S \ni v$ where $v \in L$, $v \neq 0$.

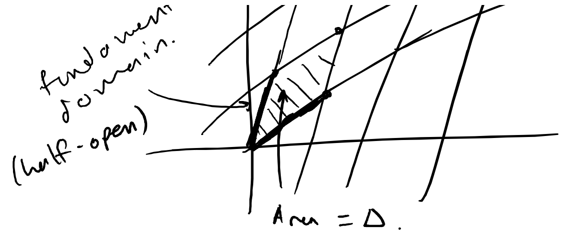
example:
$\mathbb{R}^1$



$\mathbb{Z}$ lattice. If $\mu(S \subseteq \mathbb{R}) > 1$ then $S - S$ contains an integer nonzero.

(pigeonhole principle)





$n^2\alpha$ ergodic?

(skew product on $\mathbb{T}^2$).

$$\mathbb{R}^2/\mathbb{Z}^2 = \mathbb{T}^2 \ni (x,y) \xmapsto{T} (x+\alpha,\ y+2x+\alpha) \in \mathbb{T}^2.$$

Properties of T:
1. homeomorphism
2. preserves Lebesgue measure
3. moreover, Lebesgue measure is unique T-invariant measure.

} such a map is "uniquely ergodic".

for such maps, every orbit is u.d..

$X \mapsto x+\alpha$ on $\mathbb{T}$ is also uniquely ergodic

$$(0,0) \longrightarrow (\alpha,\alpha) \longrightarrow (2\alpha, 4\alpha) \longrightarrow (3\alpha, 9\alpha) \longrightarrow (4\alpha, 16\alpha) \longrightarrow (5\alpha, 25\alpha) \longrightarrow \cdots$$

So $(n\alpha, n^2\alpha)$ is u.d. mod 1.

$$\frac{1}{N-M}\sum_{n=M}^{N-1} f(T^n x) \xrightarrow[N\to\infty]{\forall x \in X} \int_X f\,d\mu \qquad \forall f \in C(X).$$

$$(0,0) \xrightarrow{T^n} (n\alpha, n^2\alpha)$$

$$\frac{1}{N}\sum^{N-1} f(n\alpha, n^2\alpha) \longrightarrow \iint f\,d\mu$$

$$\frac{1}{N-M} \sum_{n=M}^{N-1} f(n\alpha, n^2\alpha) \longrightarrow \iint_{\mathbb{T}} f \, d\mu$$

$$\Rightarrow \quad (n\alpha, n^2\alpha) \text{ is w.d. in } \mathbb{T}^2.$$

"Integration way"

for weds. 18?