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Monday, January 7, 2019 12:40

Homework due monday, Jan 14: Ch2: 3, 9, 11, 13, 14, 15, 16, 24, 25, 26.

$$(\Omega, \mathcal{F}, P)$$

Ω ↑ sample space; $\mathcal{F}$ ↑ set of events; P ↑ Probability measure

$\mathcal{F}$ is a σ-field on Ω:

① $\mathcal{F} \subseteq \mathcal{P}(\Omega)$

② $\emptyset \in \mathcal{F}$, $\Omega \in \mathcal{F}$

③ For each $A \in \mathcal{F}$, we have $A^c = \Omega \setminus A \in \mathcal{F}$.

④ For each sequence $(A_n) \in \mathcal{F}^{\mathbb{N}}$, we have $\bigcup_{n \in \mathbb{N}} A_n \in \mathcal{F}$ (so $\bigcap_{n \in \mathbb{N}} A_n \in \mathcal{F}$ too).

(A σ-field is closed under finite unions bc $A_1 \cup \cdots \cup A_n = A_1 \cup \cdots \cup A_n \cup A_n \cup A_n \cup \cdots$

A σ-field is closed under non-vacuous finite intersections: if $n \geq 1$ then

$$A_1 \cap \cdots \cap A_n = A_1 \cap \cdots \cap A_n \cap A_n \cap \cdots$$)

(If $\mathcal{F}$ is closed under finite unions but not necessarily countable unions, it's a field).

P is a probability measure on $\mathcal{F}$:

① P is a measure on $\mathcal{F}$.

② $P(\Omega) = 1$.

To say μ is a <u>measure</u> on a σ-field $\mathcal{F}$ means:

$$\mu : \mathcal{F} \longrightarrow [0, \infty] \quad \text{s.t.}$$

① $\mu(\emptyset) = 0$

② For each disjoint sequence $(A_n) \in \mathcal{F}^{\mathbb{N}}$,

$$\mu\left(\bigcup_{n\in\mathbb{N}} A_n\right) = \sum_{n\in\mathbb{N}} \mu(A_n)$$

To say $(\Omega, \mathcal{F}, P)$ is a probability space means

① Ω is a set.

② $\mathcal{F}$ is a σ-field on Ω.

③ P is a probability measure on $\mathcal{F}$.

eg Toss a coin once. $\Omega = \{H, T\}$. $\mathcal{F} = \{\emptyset, \{H\}, \{T\}, \{H,T\}\}$. $P(A) = \frac{|A|}{|\Omega|}$.

eg Toss a coin twice. $\Omega = \{HH, HT, TH, TT\}$ (or $\{H,T\}^2$).
$\mathcal{F} = \mathcal{P}(\Omega)$. $P(A) = \frac{|A|}{|\Omega|}$.

eg Toss a coin infinitely many times. $\Omega = \{0,1\}^{\mathbb{N}}$. $\mathcal{F} = ?$. $P(A) = ?$

One viewpoint is to map $(\omega_1, \omega_2, \omega_3, \ldots)$ to $\sum_{i=1}^{\infty} \frac{\omega_i}{2^i}$.

eg Roll a die.

eg Roll two distinguishable dice. $P(\text{sum is } 8) = P(\{(4,4), (3,5), (5,3), (2,6), (6,2)\}) = \frac{5}{36}$.

eg Choose a point at random in $[0,1]^2$.

$\Omega = [0,1]^2$.

$\mathcal{F}$ = Set of Area-ble sets (?)

$P(A)$ = Area of A

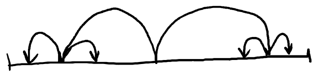
eg Choose a number at random in $[0,1]$.

$\Omega = [0,1]$.

$\mathcal{F} = ?$

$P(A)$ = length of A

The cantor set C.

 $\Rightarrow \quad |C| = |\{0,1\}^{N}| = |[0,1]|$.

$$\underbrace{S \circ R}_{\text{relation composition}} = \{(x,z) : \exists y \text{ s.t. } (x,y) \in R,\ (y,z) \in S\} \quad (S \,\&\, R \text{ relations}).$$