Lec 9/7

Friday, September 7, 2018 14:28

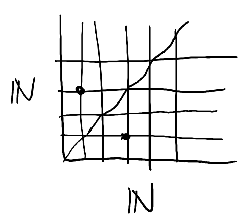
$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

Graph: $G = (V, E)$. $E \subseteq V^2$.

"for any finite coloring of a symmetric lattice, you get a symmetric lattice in one color"

Theorem (A version of Ramsey's theorem): Denote by $S^{(2)}$ the set of all 2-element subsets of S. $S^{(2)} : \{\{a,b\} \subseteq S : a \neq b\}$. Let S be a countably infinite set. $\forall$ finite coloring $S^{(2)} = \bigcup_{i=1}^{r} C_i$, one C_i contains a set of the form $R^{(2)}$, where $R \subset S$ is infinite.

ex: try to prove Erdős-Szekeres theorem using this ↑.



Erdős-Szekeres Theorem: $\forall n, \exists N(n)$ s.t. any set of N points in general position in $\mathbb{R}^2$ contains an empty convex n-gon.

ex: try using $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = \begin{pmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{pmatrix}$ to solve problems in book (And $A^m A^n = A^{m+n}$).

Hindman's Theorem: if $FS(x_n) = \bigcup_{i=1}^{r} C_i$, some $C_i \supset FS(y_n)$ for some (y_n).

Note: an infinite FS set is called IP-set.

equivalent version: if $FP(x_n) = \bigcup_{i=1}^{r} C_i$, some $C_i \supset FP(y_n)$ for some y_n.

finite product set.

multiplicative IP-set.

↳ Idempotent / Infinite-dimensional Parallelepiped

ex: prove this equivalence.

ex: Prove Jake's Corollary using VDW's Theorem.

$\forall$ finite coloring of $\mathbb{Z}^+$, any line will be AP-rich in one color.

ex: Prove Ankan's Corollary using VDW's theorem.

$\forall$ finite coloring of $\mathbb{Z}^2$, one color contains arbitrarily large grids of the form $\{a, a+d_1, \dots, a+(n-1)d_1\} \times \{b, b+d_2, \dots, b+(n-1)d_1\}$.

Li Yi Theorem: Ankan Corollary but with $d_1 = d_2$.

ex: formulate Li Yi theorem in 3 different ways.

ex: invent a ramsey-type theorem about complete graphs

ex: Let V be an infinite vector space over $\mathbb{Z}/p\mathbb{Z}$.

Invent a ramsey-type theorem about this V.