Lec 9/5

Wednesday, September 5, 2018 14:20

For Monday: ① chapter 4, ② Submit all review exercises

$FS(A)$ = finite sums of stuff in A.

Ishaan Problem: If A is an algebra (of subsets of X), let $B(x) = \bigcap_{\substack{I \in A \\ \text{s.t. } x \in I}} I$, and let $\mathcal{B} = \{B(x) : x \in X\}$.

Show there is some X, A s.t. $|\mathcal{B}| > |A|$.

Hindman's Theorem (1974): Let (x_n) be a sequence in $\mathbb{N}$. Consider a finite partition $FS(x_n) = \bigcup_{i=1}^{r} C_i$. Then at least one C_i contains a set of the form $FS(y_n)$ for some $(y_n) \subset \mathbb{N}$

Intermittent Sets: 

Freestyle Ex: give examples of nonlinear equations solvable in $FS(x_n)$

$x + y = 2z$ (arithmetic progression: $\dot{x}$ $\dot{z}$ $\dot{y}$ satisfy this).

ex: not every $FS(x_n)$ has arithmetic triples $(x + y = 2z)$ Hint: try $FS(10^n)$ or $FS(5^n)$?

$\mathbb{N} = \bigcup_{i=1}^{r} C_i \implies$ one C_i is AP-rich (Historically not correct, but Van Der Waerden).

exercise: show they are equivalent

$\mathbb{N} \supset S = \bigcup_{i=1}^{r} C_i \implies$ one C_i is AP-rich

AP-rich

exercise show they are equivalent

finitary version of VdW: $\forall \ell, r \in \mathbb{N}$, $\exists N \in \mathbb{N}$ ($N(\ell,r)$) s.t. if $\{1, \dots, N\} = \bigcup_{i=1}^{r} C_i$ then one C_i contains an AP of length ℓ.

don't submit

Ex: if $\{1,\dots,9\}$ is 2-colored then one color has length 3 AP.

Freestyle Ex: try to formulate (not equivalent) finitistic Hindman's thm

$$\text{Cantor set} = (\{0,1\}^{\mathbb{N}}, d)$$

continuous bijection