Lec 9/28

Friday, September 28, 2018 14:23

Ex: Rushil & Dennis exercises

(& Ankan)

↳ je pense no: must be uncountably many in certain bucket.

Theorem: any open set in $\mathbb{R}$ is a union of countably many disjoint intervals.

Ex Is it true that if (A_i) is a mutually indep. family then (B_i) also is where each B_i is either A_i or A_i^c?

Orthogonality / independence of functions: $\int f \cdot g = \int f \cdot \int g$.

$$\Longrightarrow \mu(A^c \cap B) = \mu(A^c)\mu(B) \text{ if } \mu(A \cap B) = \mu(A)\mu(B).$$

Countable vector space over $\mathbb{F}_p$.

$$V_{\mathbb{F}_p} = \{(a_1, a_2, \dots) : a_i \in \mathbb{F}_p, \text{ only finitely many } a_i \text{ are nonzero}\}.$$

Theorem. for any finite coloring $V_{\mathbb{F}_p} = \bigcup_{i=1}^{r} C_i$, one C_i contains arbitrarily large affine subspaces.

geometric Ramsey theorem

Ex: is it true that one C_i contains an infinite affine subspace?

(no, unless $\mathbb{F}_p$ is $\{0,1\}$).

Szemerédi analogue for $V_{\mathbb{F}_p}$: require density: use initial vector spaces as Følner sets.

↳ Any "large" set in $V_{\mathbb{F}_p}$ is ASS-rich.

as Følnersets.

Any "large" set in $V_{\mathbb{F}_p}$ is ASS-rich.

$$\bar{d}_{V_{\mathbb{F}_p}}(E) = \limsup_{n\to\infty} \frac{|E_n \cap V_n|}{|V_n|} \quad \text{where} \quad V_n = \{(a_1, \ldots, a_n, 0, \ldots) : a_i \in \mathbb{F}_p\}.$$