Lec 9/26

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Repeat exercise: do ch 4 exercises in terms of (: :).

Latin squares: each row & column contains each symbol once.

$GL_n(\mathbb{R})$

$LS_n(\mathbb{R})$

Greco-Latin squares:

a α	b β	c γ
b γ	c α	a β
c β	a γ	b α

defn.

Juxtaposition of 2 orthogonal Latin squares over 2 alphabets

Can't do it for 2x2.

Maybe can get 3 orthogonal 4 x 4 squares??

Can't do 2 for 6×6.

Reading: Latin square section (22a)

Ex: $|P(A)| > |A|$ for all sets A.

Suppose $\exists$ a bij $\begin{array}{ccc} A & \xrightarrow{f} & P(A) \\ a & \longmapsto & X_a \end{array}$. Then consider $X_n = \{a \in A \text{ s.t. } a \notin X_a\}$.

then $n \in X_n \Rightarrow n \notin X_n$, $n \notin X_n \Rightarrow n \in X_n$

Examples of natural equivalences:

① $A, B \subset X$
$A \sim B$ if $|A \triangle B| < |X|$.

② $f, g \in C[0,1]$, $f \sim g$ if $\int_0^1 f = \int_0^1 g$.

③ $A, B \subset [0,1]$ m-ble, $A \sim B$ if $\mu(A \triangle B) = 0$

Two spaces X and Y are homeomorphic $(X \overset{h}{\simeq} Y)$.
if $\exists$ bijection $f: X \longrightarrow Y$ s.t f & f^{-1} are cts.

eg riemann's sphere

Ex (bonus) this map is conformal (angle-preserving).

Ex: another metric on $\{0,1\}^{\mathbb{N}}$ is $\frac{1}{k}$ where x, y first differ in k^{th} place.

metrics d_1, d_2 are equiv. if $\forall (x_i) \subset X$, $d_1(x, x_i) \to 0$ iff $d_2(x, x_i) \to 0$.

Ex: $(\{0,1\}^{\mathbb{N}}, \rho)$ is compact

$\{0,1\}^{\mathbb{N}} = X.$

$C_0 = \{x \in X : x_1 = 0\}$

$C_1 = \{x \in X : x_1 = 1\}$

are both clopen.

(Exercise)

Ex: give an example of an open & not closed set in $\{0,1\}^{\mathbb{N}}$

$V_{\mathbb{F}_p} = \{ (a_1, a_2, \dots) : a_i \in \mathbb{F}_p, \text{ only finitely many } a_i \neq 0 \}$

$\cong \sum \mathbb{F}_p$ whatever it means.

countable vector space over $\mathbb{F}_p$.

Theorem: $\forall$ finite coloring $V_{\mathbb{F}_p} = \dot{\bigcup}_{i=1}^{r} C_i$, one C_i