Lec 9/24

Monday, September 24, 2018 14:25

Feynman books: "What do you care what other people think?"
"Tuva or bust"

Books about Erdős: "My Brain is Open"
"The man who loved only numbers"

Hardy: "Mathematician's apology"

Home exercise: what are catalan numbers.

Stuff for midterm:

- Finite fields of order p^n (existence and uniqueness).
- Fermat's little theorem
- Stirling's Formula
- Thms 3.4.1 and 3.4.2 (pgs 48 & 49)
- Formula 3.4 (pg 51).
- Lemma 3.8.2 (formula 3.12 pg 61)
- Formula 3.9 (pg 58)
- Theorem 4.3.1 (pg 71)
- Identities → problems 4.3.9 and 4.3.14 (pgs 74 ...

- Identities in problems [illegible] and [illegible] (pg 74 & 75)
- Problem 4.3.16 (b) (pg 75).
- Identities from §4.2
- Theorem 5.3.2 (pg 81)
- The Miller-Rabin test (pg 121)
- Sperner's Lemma

Reading: Chapter 7 (§1 & §2)

One can define an additive density in $\mathbb{Z}$ by taking an ARBITRARY system of intervals $I_n = \{a_n, \dots, b_n - 1\}$ s.t. $b_n - a_n \longrightarrow \infty$ and defining $\bar{d}_{(I_n)}$ as follows:

$$\bar{d}_{(I_n)}(E) = \limsup_{n \to \infty} \frac{|E \cap I_n|}{|I_n|}$$

Ex: $\bar{d}_{(I_n)}(E - t) = \bar{d}_{(I_n)}(E)$.

$$\frac{|E \cap \{1, \dots, N\}|}{N} = \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}_E(n)$$

$$\frac{|E \cap I_N|}{|I_N|} = \frac{1}{|I_N|} \sum_{n \in I_n} \mathbb{1}_E(n)$$

$$I_N = \{a_N, a_N+1, \dots, b_N - 1\}$$

$$\tilde{I}_N = \{a_N + 17, a_N + 18, \dots, b_N + 16\} \leftarrow \text{Same density.}$$

"bounded perturbations of intervals don't change density"

Ex: perturbing intervals by a fn in $o(|I_N|)$ doesn't change density.

Upper Banach density

$$d^*(E) = \limsup_{N-M \to \infty} \frac{|E \cap \{M, M+1, \dots, N-1\}|}{N-M}$$

$$d^*(E) = \sup\{\bar{d}_{(I_N)}(E) \mid (I_N) \text{ with } |I_N| \longrightarrow \infty\}$$

Ex: Can "sup" be replaced by "max"?

$d^*(E) = \alpha$ if $\forall \varepsilon > 0\ \exists M_k, N_k$ s.t.

$$\limsup_{k\to\infty} \frac{|E \cap \{M_k, M_k+1, \ldots, N_k-1\}|}{N_k - M_k} > \alpha - \varepsilon$$

and α is the least number with this property

Remark (Ex): If $d^*(E) > 0$ then for some (I_N), $|I_N| \to \infty$, $\bar{d}_{(I_N)}(E) > 0$.

$E = \bigcup I_N$. $\bar{d}_{(I_N)}(E) = 1$, but it could be that $\bar{d}(E) = 0$

eg $E = \bigcup [2^n, 2^n + n]$

ex: Can you have countably many disjoint sets (E_n) in $\mathbb{N}$ s.t. $d^*(E_n) = 1$ $\forall n$?

ex: what about with $\bar{d}$?

Yet another formulation of Szemerédi's Thm:

If $d^*(E) > 0$, then E is AP-rich.

One more:

$\forall \alpha \in (0,1)$, $\forall \ell \in \mathbb{N}$, $\exists N = N(\alpha, \ell)$ s.t. if $E \subseteq \{1, \ldots, N\}$ and $\frac{|E|}{|N|} \geq \alpha$ then E has a length-ℓ AP.

ex: prove the three form of Szemerédi's theorem are equivalent ex: prove the three form of Szemerédi's theorem are equivalent