Lec 9/12

Wednesday, September 12, 2018 14:21

Topics for Talk:

- How to guard a museum 40.
- Permanents & power of entropy 37 (too technical)
- Completing Latin Squares 36
- Shuffling Cards
- ☆ 3 famous theorems on finite sets (30)
- Tiling Rectangles
- better ☆ Pigeonhole (Sperner lemma) ← check this : pg 204
  - ↳ Brower's fixed point theorem.

Cauchy eqn $f(x+y) = f(x) + f(y)$ homomorphism $(\mathbb{R},+) \longrightarrow (\mathbb{R},+)$

↳ if f is cts then $f(x) = cx$ for some $c \in \mathbb{R}$.

ex: If f is monotone & $f(x+y) = f(x)+f(y)$

then $f(x) = cx$

Theorem: Any monotone fn $f: \mathbb{R} \longrightarrow \mathbb{R}$ has at most measure 0 of points of non-differentiability.

ex: given a countable set D, create a fn $f: \mathbb{R} \to \mathbb{R}$ which is monotone & discontinuous exactly on D.

Emile Borel's Thm almost every $x \in (0,1)$ is base-2 normal.

↳ Borel's Law of Large #s.

"Typical" continuous fn is nowhere differentiable —Banach.

↳ example (Weierstrass): $\sum a^n \sin(b^n x)$

ex: Assume Champernowne # is normal in base 10. Prove the equiv. # is normal in base 2. ⟶ not champernowne's # itself.

ex: If $A_i \subset \mathbb{R}$ $i=1,\dots$ are countably many sets of measure 0, $\bigcup_i A_i$ has measure 0.

ex: show that the classical middle thirds Cantor set is measure 0.

ex: Prove that $[0,1]$ is not of measure 0

ex: show $\exists S$ s.t. $S \cap I$ is uncountable $\forall$ interval I and $\mu(S) = 0$.

ex: The set of non-normal (base 2) #s is uncountable

ex: $C + C = [0,2]$, $C - C = [-1,1]$.