Lec 9/10

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Weierstrass Approximation Thm: polynomials (rational coefficients) are dense (uniform metric.) in $C[a,b]$.

So $C[0,1]$ is separable.

If we restrict to integer coefficients, not all intervals (& not all functions.) work, but some do.

google: phylotaxis

Ex: figure out what to do if recurrence polynomial has equal roots.

$$(x-2)^2 = x^2-4x+4 \Rightarrow f_n = 4f_{n-1} - 4f_{n-2}$$

$$f_0=0,\ f_1=1,\ f_2=4,\ f_3=12,\ f_4=32,\ \dots,\ f_n = n2^{n-1}.$$

$$(x-1+i)(x-1-i) = x^2-2x+2 \Rightarrow f_n = 2f_{n-1} - 2f_{n-2}$$

$$V = \{ (u_1, u_2, \dots) : u_{n+2} = u_{n+1} + u_n \ \forall n \geq 1 \} \quad u_i \in \mathbb{R}$$

$$T(u_1, u_2, \dots) = (u_2, u_3, \dots)$$

basis $v_1 = (0,1,1,2,\cdots)$
$v_2 = (1,0,1,1,2,\dots)$

$Tv_1 = v_1 + v_2$
$Tv_2 = v_1$

so matrix is $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

Eigenvalues?

Make more matrices using other basis $\quad v_1 = (a, b, a+b, \dots)$
$\quad v_2 = (c, d, c+d, \dots)$

$$Tv_1 = (b, a+b, a+2b, \dots) = \alpha(a, b, \dots) + \beta(c, d, \dots)$$

$$Tv_2 =$$

$$\alpha a + \beta c = b$$
$$\alpha b + \beta d = a+b$$

calculate A

Ex: $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}^n = A^{-1} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}^n A$, so $A \begin{pmatrix} u_{n+2} & u_{n+1} \\ u_{n+1} & u_n \end{pmatrix} A^{-1} = \begin{pmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{pmatrix}$.

use this to obtain $F_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right)$.

Ex: for $f_{n+3} = f_n + f_{n+1} + f_{n+2}$, we get $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

(fair) coin tossing sequence $x \in \{0,1\}^{\mathbb{N}}$.

Claim: with probability 1, x contains equal # of 0s and 1s.

Def'n a 0-1 sequence is called normal if $\forall k$, length-k finite word w, the frequency of appearances of w in x equals $\frac{1}{2^k}$.

[illegible]