Lec 8/24

Friday, August 24, 2018 14:21

Reading: Ch 1 + 1/2 of Ch 2

Submit Book Problems: 1.3.(2,3,4), 1.4.(1,2), 1.8.(4,5,6,7,20,21,23,29,33)

$$A,\ BA,\ BBA,\ \dots,\ \underbrace{B\cdots B}_{n}A,\ \dots,\ \underbrace{B\dots}_{\infty}$$

$(p+q=1,\ p>0,\ q>0)$

$$p,\ qp,\ qqp,\ \dots,\ \underbrace{q\cdots q}_{n}p,\ \dots,\ \underbrace{q\dots}_{\infty}$$

$$p \quad qp \quad q^2p \quad \dots \quad q^n p \quad \dots \quad 0$$

$$\sum_{n=0}^{\infty} pq^n = 1 \quad \Rightarrow \quad \sum_{n=0}^{\infty} q^n = \frac{1}{p} = \frac{1}{1-q}$$

Now: finish w/ AA.

exercise: Prove that # of words of length n are fibonacci #s

AA, BAA, ABAA, BBAA, ABBAA, BABAA, BBBAA, ...

	1	1	2	3	5?	8?	...
P	$\frac{1}{2^2}$	$\frac{1}{2^3}$	$\frac{2}{2^4}$	$\frac{3}{2^5}$	$\frac{5}{2^6}$	$\frac{8}{2^7}$	...

$$\sum_{n=0}^{\infty} \frac{F_n}{2^{n+1}} = 1 \quad (*)$$

exercise: use F_n definition to prove this

exercise: generalize to p, q (not necessarily $p=q=\frac{1}{2}$)

exercise: prove the set of infinite outcomes is uncountable.

Think about 3 As

exercise: Try to justify (*) probabilistically, given above

How many subsets of $\{1,\dots,n\}$? $2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k}$

$\{0,1\}^n \equiv$ 0-1 seqs of length $n \equiv$ fns $\{1,\dots,n\} \to \{0,1\}$. Size 2^n.

$\{0,1\}^{\mathbb{N}}$ = infinite (one-sided) 0-1 sequences $\approx \mathcal{P}(\mathbb{N})$.

exercise: Characterize those $x \in [0,1]$ w/ two n-ary expansions

$$|1_A - 1_B| = 1_{A \triangle B = (A \cup B) \setminus (A \cap B)}$$

Algebra of sets is a set of sets closed under $\cap$, $\cup$, complement.

ex: $\mathcal{P}(X)$, $\{\emptyset, X\}$

finite set, cardinality n.

ex: let $\mathcal{A}$ be an algebra of subsets of X. prove $|\mathcal{A}| = 2^k$ for some $k \in \{1, \ldots, n\}$

ex: if X is infinite and $\mathcal{A}$ is finite then still $|\mathcal{A}| = 2^k$ for some $k \in \mathbb{N}$.

utilities problem: exercise: prove it's impossible

Freestyle: google Königsburg bridges problem.