Lec 12/3

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Sharkovski's Order on $\mathbb{N}$.

Claim: in $\mathbb{R}^2$, the set of extreme points of a convex body is closed

Challenge: is it true in $\mathbb{R}^3$?

1D - HJ $\Longrightarrow$ 2D - HJ by just replacing alphabet A by A^2.

IP VdW Theorem: If $\mathbb{N} = \bigcup_{i=1}^{r} C_i$ and $A = FS(n_i)_{i=1}^{\infty}$, one of C_i contains arbitrarily long $\{x, x+d, \cdots, x+nd\}$ with $d \in A$.

IP Szemeredi: If $E \subset \mathbb{N}$ and $d^*(A) > 0$, then $\forall k \in \mathbb{N}$, $\{d : \exists x : \{x, x+d, \dots, x+kd\} \subset E\}$ is IP*.

Ex: Also, $\{d : \bar{d}(E \cap (E-d) \cap \dots \cap (E-kd)) > 0\}$ is IP*

Show this from IP Sz

If A, B are IP* then $A \cap B$ are IP*.

Since $\mathbb{N} \setminus (A \cap B) = (\mathbb{N} \setminus A) \cup (\mathbb{N} \setminus B)$. (Apply Hindman's Theorem).

Remark: any IP* set A intersects any IP set C along a sub-IP of C.

Ex: HJ $\Longrightarrow$ IP vdW

"Thick sets have shifts of arbitrarily large "neighborhoods of 0" ".

$\{n : \|n^2\alpha\| < \varepsilon\}$ is IP* using 2D-IP van der Waerden

(recall proof that $n^2\alpha$ is dense).

Color (n,m) by the ε-subinterval that $nm\alpha$ falls into (mod 1)

and $(m+d)(n+d) - n(m+d) - m(n+d) + nm = d^2$

Corollary: $\forall$ polynomial $p(t) \in \mathbb{R}[t]$ with $p(0)=0$, the set $\{n : \|p(n)\| < \varepsilon\}$ is IP*

Proof: $\{n : \|n^k\alpha\| < \frac{\varepsilon}{m}\}$ is IP*. Take intersections ($m = \deg p$).

Any set $A \subset \mathbb{N}$ with $\bar{d}(A) > 0$ contains $\forall k$ "many" sets of the form

$Q(t, x_1, \ldots, x_n) = t + \{\sum \varepsilon_i x_i : \varepsilon_i = 0 \text{ or } 1\}$.

Fact: $\forall t \in \mathbb{R}$, $\exists x, y \in \mathbb{R}$ s.t. x mod 1 & y mod 1 are base-2 normal & $t = x - y$.

pf Co-null sets intersect. $N \cap (N+t) \neq \emptyset$.