Lec 11/7

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Call $\{c(a+di)^j : 0 \le i, j < k\}$ a G-A progression of length k.

If $\mathbb{N} = \bigcup_{i=1}^{r} C_i$, is it true that one C_i is GAP rich?

Yes: use Hales-Jewett theorem.

Any multiplicatively large set is GAP rich too.

Ex: there is no field of triples in $\mathbb{R}$

$$\mathbb{H} = \{a + bi + cj + dk : a, b, c, d \in \mathbb{R}\}$$

w/ $i^2 = j^2 = k^2 = -1$,

$$\begin{tikzcd} & i \arrow[dr] & \\ k \arrow[ur] & & j \arrow[ll] \end{tikzcd}$$

$$a + bi + cj + dk = (a+bi) + (c+di)j$$

Shortcut: $a + bi \approx \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

$$z + wj \approx \begin{pmatrix} z & w \\ -\bar{w} & \bar{z} \end{pmatrix}$$

determinant $= |z|^2 + |w|^2 = a^2 + b^2 + c^2 + d^2$

Euler's Theorem: if x & y are sums of 4 squares, $x \cdot y$ is a sum of 4 squares.

Pf: & check that $\begin{pmatrix} z_1 & w_1 \\ -\bar{w}_1 & \bar{z}_1 \end{pmatrix} \begin{pmatrix} z_2 & w_2 \\ \ldots & \ldots \end{pmatrix} = \begin{pmatrix} z_3 & w_3 \\ \ldots & \ldots \end{pmatrix}$.

$-w_2 \quad z_2 \qquad -w_3 \quad z_3$

$\overline{a+bi+cj+dk} = a-bi-cj-dk$

Cayley's Octavas

<u>Frobenius</u>: the only finite-dimensional division algebras over $\mathbb{R}$ are: $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$.

For Midterm:

<u>Ch9</u>: Just definitions (Hamiltonian Cycles, Traveling Salesman Problem).

<u>Ch10</u>: Important Stuff (König-Egerwary thm, Dilworth Thm, Menger thm)

↳ thm 10.1.1 (König) Know to prove

10.3.1 (marriage) know to prove.

<u>Ch11</u>: Thm 11.2.1 (know to prove). Convexity stuff

<u>Ch12</u>: Thm 12.2.2 (use euler's formula to prove)

thm 12.2.1

<u>Ch14</u>: Steiner Systems (14.4.1, 14.4.2)

Magic / Latin Squares

Codes

<u>Class</u> formulated Stuff (Hales-Jewett, ergodic Theorem, Särkozy thm)

prove (von Neumann - Birkhoff thm on bistochastic matrices)

(a.e. # is normal)

Ergodic Szemerédi theorem:

$\forall$ probability space X, any probability-preserving map $T: X \longrightarrow X$

any $A \subset X$ with $\mu(A) > 0$, and any $k \in \mathbb{N}$, $\exists\, n \in \mathbb{N}$ s.t.

$$\mu(\underbrace{A \cap T^{-n}A}_{\text{Poincare recurrence theorem}} \cap T^{-2n}A \cap \cdots \cap T^{-kn}A) > 0$$

Poincare recurrence $\rightarrow$ If $\forall\, A \subset X$ with $\mu(A) > 0$ $\exists\, n \in \mathbb{N}$ s.t. $\mu(A \cap T^{-n}A) > 0$

corollary $\rightarrow$ then for a.e. $x \in A$, $\exists\, n \in \mathbb{N}$ s.t. $T^n x \in A$.

exercise

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