Lec 11/5

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Given a continuous map $\phi : X \times Y \longrightarrow \mathbb{R}$, Say X is ϕ-achieved on Y if $\exists \alpha \in \mathbb{R}$ s.t. $\forall x \in X$, $\exists y \in Y$ s.t. $\phi(x,y) = \alpha$.

Ex: Show that if X and Y are compact and connected & $\phi : X \times Y \longrightarrow \mathbb{R}$ is continuous then either X is ϕ-achieved on Y or Y is ϕ-achieved on X.

Ex: $\bar{d}(P) = 0$ (follows from a weak form of the PNT: $p_n \sim n \log n$).

that $\frac{p_n}{n \log n}$ is bounded

Erdős's Question / Conjecture: (OPEN!!!)

If $E = \{n_1 < n_2 < \cdots\}$ and $\sum_{i=1}^{\infty} \frac{1}{n_i} = \infty$. is it true that E is AP-rich?

Ex: $\sum \frac{1}{n \log n \log\log n} = \infty$

Ex: Is it true that $P-1$ is GP rich? (VB thinks yes)

Ex: Show that $P-1$ contains #s w/ arbitrarily many divisors

Ex: Is $(P-1) \cap$ Square-free infinite? (VB thinks yes)

Ex: Let $P(x_1, \ldots, x_n)$ be a nonzero polynomial which vanishes if $x_i = x_j$ $\forall i \neq j$. What is the minimum number of terms of P.

Ex: What is the maximal size of an abelian subgroup of S_n

Ex: Show that $\exists$ $(r_n) \subset \mathbb{Q} \cap (0,1)$ which is u.d. & s.t. $\{r_n : n \in \mathbb{N}\} = \mathbb{Q} \cap (0,1)$

Ex: Show (x_n) is dense in $[0,1)$ iff $\exists$ bijection $f: \mathbb{N} \longrightarrow \mathbb{N}$ s.t. $(x_{f(n)})$ is u.d. (von Neumann's rearrangement theorem)

<u>Sárközy's theorem</u>: If $A \subset \mathbb{N}$, $\bar{d}(A) > 0$, then $A - A$ contains
(~ 1975) ∞-many squares.

$(A - A) \cap (B - B) \ni n^2$??

SANOV's group