Lec 11/30

Friday, November 30, 2018 14:24

Sum-free set: $x, y \in A \Rightarrow x+y \notin A$.

A_n = even permutations of $\{1, \dots, n\}$.

$A = \bigcup_{n=2}^{\infty} A_n$ = even finite permutations of $\mathbb{N}$.

$E \subset A$, $\bar{d}(E) = \limsup_{n \to \infty} \frac{|E \cap A_n|}{|A_n|}$

$E \subset \sum_{1}^{\infty} \mathbb{Z}/p\mathbb{Z}$, $\bar{d}(E) = \limsup_{n \to \infty} \frac{|E \cap \sum_{1}^{n} \mathbb{Z}/p\mathbb{Z}|}{p^n}$

Ex: show $\bar{d}$ is invariant under a single group operation

A subgroup $S \leq G$ is syndetic iff it has finite index.

In A, every "large" set has Schur property.

A is a simple group.

Ex: if $\bar{d}(A) > \frac{1}{2}$, $\bar{d}(A \cap A - x) > 0$ $\forall x$.

This is true in any group which has an analogue of $\bar{d}$.

Ex: Is $\{(n, m) \in \mathbb{N}^2 : (n, m) = 1\}$ syndetic?

Let a sphere $S_n \subset \overset{\text{semigroup on } a,b}{\overset{\shortparallel}{\langle a,b \rangle}}$ be the set of words of length n.
The ball B_n is all words of length $\leq n$.

Ex: $\frac{|S_n|}{|B_n|} \longrightarrow ?$ guess: $\frac{1}{2}$.

$|S_n| = 2^n$. $\sum_{0}^{N-1} 2^n = 2^N - 1$

in F_2: $|S_n| = 3|S_{n-1}|$, $|S_1| = 4$.

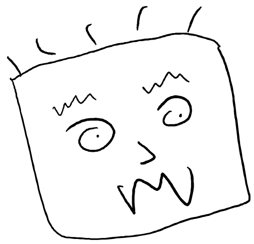
① A set $A \subseteq \mathbb{R}^2$ has measure-0 if $A \subset \bigcup S_i$ (square) w/ $\sum \mu(S_i) < \varepsilon$.

② ——— " ——— $A \subset \bigcup R_i$ (rectangles) w/ $\sum \mu(R_i) < \varepsilon$

③ ——— " ——— $A \subset \bigcup D_i$ (disks) w/ $\sum \mu(D_i) < \varepsilon$

Fact: non-normal #s have measure zero

typical



Koksma

Theorem: for a.e. $x>1$, the sequence $x^n \bmod 1$ is u.d.

Remarks: there are uncountably many $x > 1$ for which $x^n \bmod 1$ is not 0.

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