Lec 11/26

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Construct $(a_n) \subset \mathbb{N}$ s.t. $\underbrace{\{a_n\alpha \bmod 1 : n \in \mathbb{N}\}}$ has uncountably many accumulation points but is nowhere dense.

a_n = next highest integer s.t. $d(a_n\alpha, C) < \frac{1}{n}$. (cantor set)

does it work?

$a_n = 3^n$, α is $\{0,2\}$-normal

Ex: Look at $n!e \bmod 1$

Ex: if $\bar{d}(A) > 0$, $|(A-A) \cap FS(10^n)| = \infty$, and in fact $A-A$ contains a sub-IP of $FS(10^n)$.

Furstenburg Correspondence Principle:

$E \subset \mathbb{N}$, $\bar{d}(E) > 0 \implies \exists\ (X, \mathcal{B}, \mu, T)$, $\exists\ A, B \in \mathcal{B}$

s.t. $\mu(A) = \bar{d}(E)$ and $\forall\ n_1, \dots, n_k \in \mathbb{N}$,

$$\bar{d}(E \cap (E-n_1) \cap \dots \cap (E-n_k)) \geq \mu(A \cap T^{-n_1}A \cap \dots \cap T^{-n_k}A).$$

Fact: If $A_n \in \mathcal{B}$, $\mu(A_n) = a > 0$ $\forall n \in \mathbb{N}$, then $\forall k$

$\exists n_0, d$ s.t. $\mu(A_{n_0} \cap A_{n_0+d} \cap \dots \cap A_{n_0+kd}) > 0$.

Challenge: give a "purer" combinatorial proof of #6.

for #8: Any pws set contains a shift of an IP-set.

rank of syndeticity $= \frac{1}{10} \Rightarrow$ $\bigcup$ 11 shifts is thick.

· Any thick set contains an IP set, and by Hindman's Theorem, one of the shifts contains an IP set.

Write up: My solution to #8

Ex: prove piecewise syndeticity is partition regular (without ultrafilters).

Review: derangements, VS over $\mathbb{F}_p$ and q-coefficients (from cameron).

Exercise: $\forall \varepsilon > 0$, $\exists n : \|n^2\alpha\| < \varepsilon$,

$\forall \varepsilon > 0$, $\exists n : n^2\alpha \bmod 1 < \varepsilon$

Show that if $\exists n : n^2\alpha \bmod 1$ then $\exists n : n^2\alpha \bmod 1 > 1-\varepsilon$.