Lec 11/14

Wednesday, November 14, 2018 14:22

Smael's Horseshoe

Bistochastic Matrices form a semigroup

Group: Locally compact. Haar measure.

$T: X \longrightarrow X$ gives $\mathbb{Z}$-action $x \longmapsto T^n x$.

$T^n: f(x) \longmapsto f(T^n x)$ So $\mathbb{Z}$ action on functions on X.

$\frac{1}{N} \sum f(T^n x) \xrightarrow{a.e.} \int f$

$\frac{1}{N} \sum T^n f \xrightarrow{a.e.} \int f$

$T(\alpha f + \beta g) = \alpha T f + \beta T g$

group acting, counting, combinatorics.

Smael's horseshoe - governed by dynamics on Cantor set.

$A \in p+q \iff \{n : (A-n) \in p\} \in q$.

by Ellis Lemma, $\exists\, p = p+p$.

$\mathbb{N} = \bigcup_{i=1}^{r} C_i \Rightarrow$ one $C_i = C$ is p-large (i.e. $C \in p$).

$C \in p\text{-}p+p \Rightarrow \{n : (C-n) \in p\} \in p$. (since $C \cap \{n : (c-n) \in p\} \in p$)

So p-many shifts of C are p-large. So $\exists n_1 \in C$ s.t. $C - n_1 \in p$. So $C \cap C - n_1 \in p\text{-}p+p$

continue, letting

$B_1 = C \cap C - n_1$

and finding $n_2 \in B_1 = C \cap C - n_1$

$B_2 = B_1 \cap B_1 - n_2 \in p = p+p$.

etc.

$FS(n_i) \subset C$. ←

Ex: $\forall \varepsilon > 0$, $\exists$ A w/ $\bar{d}(A) > 1-\varepsilon$ s.t. A contains no shift of an IP set.

↳ find proof of Ellis Lemma online (VB survey).

Midterm Stuff from Handouts: (Read Derangement stuff, any interesting things can be asked (Bell #s, Derangements, Catalan #s)

- Automorphism groups/permutation groups
  - ↳ 14.3 orbits & transitivity
  - Lagrange Thm
  - 14.4.2 Schreier's Lemma

- Generating Functions
  - ↳ Definitions of Generating functions (ordinary & exponential)
  - ↳ Examples: eg 8.3 on p. 155. (problems like this)
  - ↳ products of generating fn's & interpretation (Thm 8.5 pg 156)
  - ↳ examples: 8.7 & 8.8.
  - ↳ Catalan $C(x) = \frac{1-\sqrt{1-4x}}{2x}$ pg 161-162
  - ↳ more examples w/ exponential GF.

- Matching, Covering & Packing.
  - 3 or 4 theorems which are equivalent (max-min)
  - find 8 applications of marriage lemma

König, Dilworth, Marriage thm, Hall, Menger, König - Egervary -

find matrix version of all of these.

TO know → Hall thm. Defn of Stable matching. Something about path covers.

Know: König-Egervary & Bistochastic matrix theorem

Know: Carathéodory theorem. Browse Convexity Packet too