Lec 10/29

Monday, October 29, 2018 14:21

Can $\mathbb{N} = C_1 \cup C_2$ with $\bar{d}(C_1) = 0$ and $\bar{d}_x(c_2) = 0$?

$\downarrow$

$\limsup_{n \to \infty} \frac{|C_2 \cap F_n|}{|F_n|}$

where $F_N = \{ a_N\, p_1^{i_1} p_2^{i_2} \cdots p_N^{i_N} ,\ 0 \le i_j \le N \}$

Ex:

Note: $\bar{d}_x(\text{square-free}) = 0$

So $\bar{d}_x(\text{k-th power free}) = 0$

So we can get within ε. (since $\bar{d}(\text{kth power-free}) \longrightarrow 1$ as $k \to \infty$)

Ex:

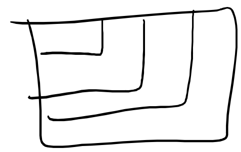
(a) Let A, B be properties of subsets of $\{1, \dots, n\}$ (i.e. $A, B \subseteq P(\{1, \dots, n\})$) so that $A^* = B$ and $B^* = A$. Prove that $|A| + |B| = 2^n$

(b) Prove that $\exists\, c > 0$ s.t. $\forall n\ \exists$ at least $c \cdot n!$ properties P on subsets of $\{1, \dots, n\}$ for which $P^* = P$.

(c) What is the true asymptotic for (b)? (Ankan believes it is larger than $n!$)

A matrix is Unimodal if every square submatrix has determinant 0, 1, or -1.

Sylvester's Criterion for positive definiteness of matrix: every principal square submatrix has positive determinant



Ex: Let (X, d) be a connected compact metric space. Prove that there exists $\alpha > 0$ such that $\forall\, x_1, \dots, x_n \in X$, $\exists\, x \in X$ s.t.

$$\frac{d(x, x_1) + \cdots + d(x, x_n)}{n} = \alpha$$

Midterm (proof)

Birkhoff - von Neumann: any bistochastic matrix is a convex combination of permutators.

$\Uparrow$

König - Egervary theorem: if A is an $n \times n$ 0-1 matrix then the minimum # of lines needed to cover all 1^s = the maximum size of "independent" set of 1^s.

$A = (a_{ij})$. ← bistochastic. Observation: you need n lines to kill all non-zero elements.

Suppose m lines $\ell_1, \dots, \ell_m$ will do. then $\sum_{\substack{a_{ij} \neq 0 \\ a_{ij} \in \ell_k}} a_{ij} \le m < n$

So by K-E $\exists$ P a permutator and $c > 0$ s.t.

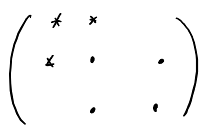
$A - cP$ has nonnegative elements and

has one more zero, and sum of elements

over any row or column is $1-c$ (multiply by $\frac{1}{1-c}$ to

get back to a bistochastic matrix).

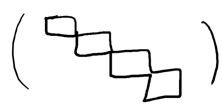
How to prove B-vN without K-E?



Can assume $*$'s are $\neq 0$.

also assume no 1's.

In general, can create



Reading
Three famous theorems on finite sets (Especially marriage thm).

Hales-Jewett theorem (Polymath 1)

$\hookrightarrow$ in high-enough dimension tic-tac-toe, there is always a winner.

Let $A \cup \{*\}$ be a finite alphabet. a finite word over $A \cup \{*\}$

$A = \{a_1, a_2, a_3\}$

is nontrivial (for our purposes) if it contains t.

Def A combinatorial line : $\{ \overbrace{W(t)}^{\text{variable words.}} : t \in A \}$

nontrivial word containing t which is viewed as a variable

A^n = all words of length n.

Hales-Jewett If (the size of) A is fixed, then $\forall r \in \mathbb{N}$ $\exists n \in \mathbb{N}$ s.t.

if $A^n = \bigcup_{i=}^{r} C_i$ then at least one C_i contains a combinatorial line.

Ex: show $HJ \implies vdW$

$HJ \implies V_{\mathbb{F}_p} = \bigcup_{i=1}^{r} C_i \Rightarrow$ one C_i is shift of subspace-rich