Lec 10/26

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See how Neil Hindman's defn of d^* works

if $A \in \{0,1\}^{\mathbb{N}}$ is normal, $\Omega(A) = \overline{\{T^n A : n \in \mathbb{N}\}} = \{0,1\}^{\mathbb{N}}$

T: shift op

$\ell^\infty(\mathbb{Z})$ = bdd fns on $\mathbb{Z}$. $\ell^2(\mathbb{Z}) = \{f : \mathbb{Z} \to \mathbb{R} \mid \sum_{n \in \mathbb{Z}} f(n) < \infty\}$

$\|f\| = \sup_{n \in \mathbb{Z}} |f(n)|$

$1_A \in \ell^\infty(\mathbb{Z})$. $1_A(x-t) = 1_{A+t}(x)$

$P(\mathbb{N}) \longleftrightarrow$ properties ultrafilters

Ex: if $P \subseteq P(\mathbb{N})$ is an ultrafilter then $P = P^*$

Ex: show $P^* = P^{***}$ for any property P.

Ex: Show $\{n : \|n^2\alpha\| < \varepsilon\}$ is not Δ^* (but it is IP^*)

($\|\cdot\|$: distance to closest integer)

Ex: Define syndeticity & thickness in general semigroup

Show $\text{Syndetic}^* = \text{Thick}$, $\text{Thick}^* = \text{Syndetic}$.

Bistochastic matrices are convex combinations of permutations.

Space of bistochastic matrices is a convex set.

Ex: Let B be a convex body in $\mathbb{R}^2$ (closed, bounded, w/ nonempty interior, convex).

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then any $x \in B$ is a convex combination of at most 3 <u>extreme</u> points → boundary points

Extreme points are those which are <u>not</u> nontrivial convex combinations of others.

improvement: $2 \longrightarrow k$, $3 \longrightarrow k+1$

Ex: show that extreme points of set of Bistochastic matrices are exactly the permutations.

A line in a matrix is a row or column

<u>Lemma:</u> Minimum # of lines needed to cross out all 1s in a 0-1 matrix $=$ max size of independent (diff row & column) set.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

Ex: prove it

$T : [0,1] \longrightarrow [0,1]$ is measure-preserving

if $\forall$ m-ble $A \subset [0,1]$, $\mu(T^{-1}A) = \mu(A)$

eg $x \longmapsto 3x \mod 1$

$x \longmapsto x + \alpha \mod 1$