Lec 10/19

Friday, October 19, 2018 14:24

$\mu(\bigcup_{i=1}^{n} C_i) = \sum_{i=1}^{n} \mu(C_i)$

An <u>ultrafilter</u> on $\mathbb{N}$ is (for us) any 0-1 valued $\overbrace{\text{finitely additive}}$ probability measure on $\mathcal{P}(\mathbb{N})$ (space of ultrafilters $= \beta\mathbb{N}$)

$\mathbb{N} \hookrightarrow \beta\mathbb{N}$

$n \longmapsto \mu_n(A) = \begin{cases} 1 & A \ni n \\ 0 & A \not\ni n \end{cases}$ (i.e. $\mathbb{N} \subset \beta\mathbb{N}$)

read: Baumgartner's proof of Hindman's Theorem

<u>Thick</u> iff $\exists n$ s.t. $S - n \supset F$ $\forall F \in \mathcal{F}$

<u>Multiplicatively Thick</u> iff $\forall F \in \mathcal{F}$ $\exists x$ s.t. $S/x \supset F$.

<u>Ex</u>: Prove or disprove. An IP^* set is multiplicatively thick.

Szemeredi: If $A \subseteq \mathbb{N}$ is large ($d^*(A) > 0$ or $\bar{d}(A) > 0$) then A contains an affine image of any finite set.

Ex: if $\lim_{n\to\infty} \frac{a_n}{2^n} < 1$ is it true that $\bar{d}(A) > 0$?

If a set has $d^*(A) = 1$ then A contains isometric shifts of any finite set. and a $FS(n_i)_{i=1}^{\infty}$.

$S \subseteq G$ is syndetic if $\exists$ finite $F \subset G$ s.t. $G = \bigcup_{x \in F} S/x$

$S \subseteq G$ is syndetic if $\exists$ finite $F \subset G$ s.t. $G = \bigcup_{x \in F} S/x$

$S \subseteq G$ is thick if $\forall$ finite $F \subset G$, $\exists x \in G$ s.t. $S/x \supset F$.

Ex: prove any thick set in $(\mathbb{N}, \times)$ contains a multiplicative IP set $(FP(n_i)_{i=1}^{\infty})$.

two possible defns of IP set in general (non-commutative) semigroup:

① $\{ x_{i_1} \cdot x_{i_2} \cdot \dots \cdot x_{i_k} : k \in \mathbb{N},\ i_1 < i_2 < \dots < i_k \}$

② $\{ x_{i_1} \cdot x_{i_2} \cdot \dots \cdot x_{i_k} : k \in \mathbb{N},\ i_1 > i_2 > \dots > i_k \}$

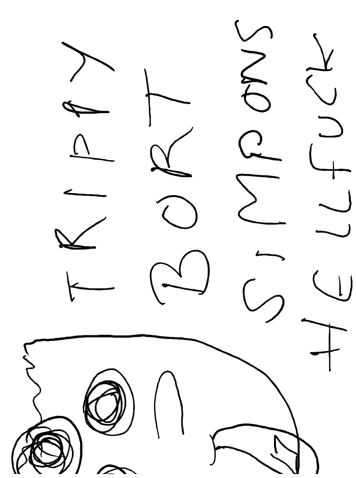
Ex: In a more naive way, one can consider sets of all possible products (in any order). Prove Hindman's theorem is not true for this type.

Hint: free group.

F_2 is paradoxical:

$A^+ = a F_2$, $\quad b^n A^+ \cap b^m A^+ = \emptyset \quad \forall n \neq m$

if $d(A^+) = c > 0$ then contradiction.



if $d(A^+) = c > 0$ then contradiction.