Lec 10/17

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Theorem: Any chastic bisto matrix is a convex combination of Permutators

(Von Neumann / Birkhoff theorem)

Find tiling of $\mathbb{N}$ by finitely many APs w/ all different differences

Read first bit of generatingfunctionology by H. Wilf

Ex: If $G \leq V_{\mathbb{F}_p}$ is infinite then $G \overset{\text{gp. isomorphism}}{\cong} V_{\mathbb{F}_p}$.

Ex: $V_{\mathbb{F}_p}$ only has countably many finite subgroups

Any abelian group is a commutative module over $\mathbb{Z}$.

Lemma: assume $A \subset \mathbb{N}$ and $\bar{d}(A) = 1$. then A is GP-rich.

both are exercises

Ex: Is it true that $\forall \varepsilon > 0$, $\exists A \subset \mathbb{N}$ w. $\bar{d}(A) > 1 - \varepsilon$ and A is not GP-rich.

Ex. if $\mathbb{N} = \bigcup_{i=1}^{r} C_i$, $\mathbb{N} = \Big(\underbrace{\bigcup_{i=1}^{r_0} C_i}_{\text{positive density guys}}\Big) \cup \Big(\underbrace{\bigcup_{i=r_0+1}^{r} C_i}_{\text{zero density guys}}\Big)$ where $\underbrace{\bar{d}\Big(\bigcup_{i=1}^{r_0} C_i\Big) = 1}$ $\longrightarrow$ one C_i is GP-rich by VdW

Lemma: Any set A with $\bar{d}(A) = 1$ is AP-rich   All are AP-rich by Sz.

$\hookrightarrow$ Hints. $\bar{d}(A \cap A/n) = 1$, etc.

$\downarrow$

$\bar{d}\big((A \cap A/n) \cap (A \cap A/n)/m\big)$ maybe

$\bar{d}(A \cap (A - n)) = 1$, etc.

Book: do exercises from 11 & 12.

$(n, \lfloor n\alpha \rfloor) \stackrel{?}{=} 1$ density of $\frac{6}{\pi^2}$ for both???

$(n, \lfloor n^c \rfloor) \stackrel{?}{=} 1$